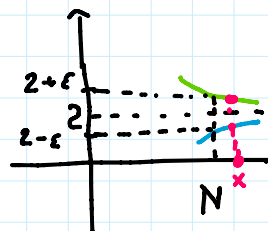


APPELLO 22. 01. 2025

VERIFICA $\lim_{x \rightarrow +\infty} \frac{2x + \sin x}{x} = 2$



DEF: $\forall \epsilon > 0 \exists N \mid |f(x) - l| < \epsilon \quad \forall x > N$

$$\left| \frac{2x + \sin x}{x} - 2 \right| < \epsilon$$

$$\left| \frac{\cancel{2x} + \sin x - \cancel{2x}}{x} \right| < \epsilon$$

$$\left| \frac{\sin x}{x} \right| \leq \left| \frac{1}{x} \right|$$

$x \rightarrow +\infty$ ALLORA $x \geq 0 \rightarrow |x| = x$

$$\left| \frac{\sin x}{x} \right| \leq \frac{1}{x}$$

SE RISOLVO $\frac{1}{x} < \epsilon$ ANCHE $\left| \frac{\sin x}{x} \right| < \epsilon$

ANCHE $\left| \frac{2x + \sin x}{x} - 2 \right| < \epsilon$

$$\frac{1}{x} < \epsilon \quad x > \frac{1}{\epsilon}$$

i.e. SE $\epsilon = 0.00000001$

$\frac{1}{\epsilon} = 100000000 \quad x > \underline{100000000}$
↓
N

BASTA SCEGLIERE $N = \frac{1}{\epsilon}$

22.6.22

$$\lim_{x \rightarrow 1} \frac{x+3}{3x+1} = 1 \quad \forall \varepsilon > 0 \exists \delta : |x - x_0| < \delta \rightarrow |f(x) - l| < \varepsilon$$

$$\left| \frac{x+3}{3x+1} - 1 \right| < \varepsilon \quad \left| \frac{x+3-3x-1}{3x+1} \right| < \varepsilon$$

$$\left| \frac{2-2x}{3x+1} \right| < \varepsilon$$

$$\left| \frac{2-2x}{3x+1} \right|_{x \geq 0} = \frac{|2-2x|}{3x+1} = \frac{|2x-2|}{3x+1}$$

$$|3x+1| = 3x+1$$

$$\frac{2|x-1|}{3x+1} < \frac{2|x-1|}{3 \cdot 0 + 1} \text{ cioè } 2|x-1|$$

ALLORA $\left| \frac{x+3}{3x+1} - 1 \right| < 2|x-1|$

$$\text{SE } 2|x-1| < \varepsilon \quad \text{ANCHE } \left| \frac{x+3}{3x+1} - 1 \right| < \varepsilon$$

$$\Rightarrow |x-1| < \frac{\varepsilon}{2} \quad \delta = \frac{\varepsilon}{2}$$

$$-\frac{\varepsilon}{2} < x-1 < \frac{\varepsilon}{2}$$

$$1 - \frac{\varepsilon}{2} < x < 1 + \frac{\varepsilon}{2}$$

$$\frac{1-\frac{\varepsilon}{2}}{\quad} \quad 1 \quad \frac{1+\frac{\varepsilon}{2}}{\quad}$$