

APPELLO 12.01.2017

NON VUOL DIRE CLASSIFICARE

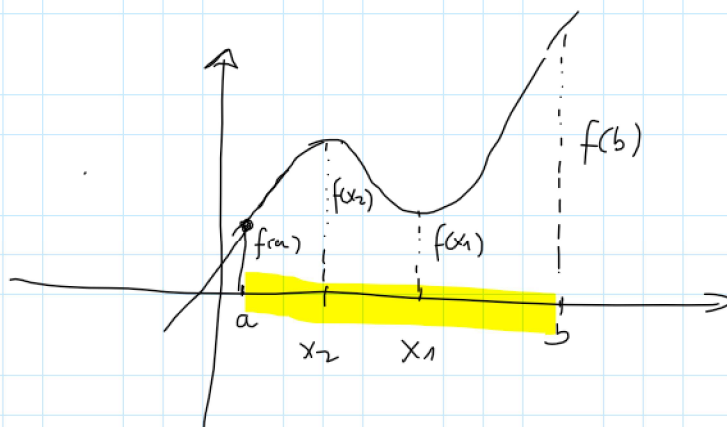
1) INDIVIDUARE MASSIMI E MINIMI LOCALI E GLOBALI

01 $f(x) = \frac{x}{x^2+4}$ IN $[1, 5]$ $\hookrightarrow f'(x) = 0$

2) SCRIVERE L'EQ. DELLA RETTA TG A $f(x)$ NEL SUO PUNTO DI MINIMO GLOBALE

3) DISEGNARE IL GRAFICO DI $f(x)$ NEL SUO DOMINIO

MASSIMI E MINIMI LOCALI E GLOBALI



IN $[a, b]$ IL VALORE PIÙ PICCOLO È $f(a)$
IL VALORE " GRANDE È $f(b)$

$x_1 \rightarrow f(x_1)$ MAX / MINIMI LOCALI $f'(x) = 0$ $\begin{matrix} x_1 \\ x_2 \end{matrix}$
 $x_2 \rightarrow f(x_2)$ TEOREMA FERMAT

COME PROCEDO

1) CALCOLO $f(a)$ e $f(b)$

$$f(1) = \frac{1}{1+4} = \frac{1}{5}$$

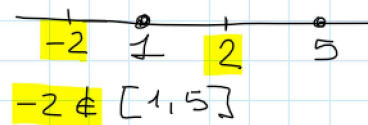
$$f(5) = \frac{5}{25+4} = \frac{5}{29}$$

$$2) f(x) = \frac{x}{x^2+4}$$

$$f'(x) = \frac{1 \cdot (x^2+4) - x(2x)}{(x^2+4)^2} = 0 \quad (\text{FERMAT})$$

$$x^2 + 4 - 2x^2 = 0$$

$$-x^2 + 4 = 0 \quad x^2 = 4 \quad x = \pm 2$$



$$x=2 \rightarrow f(2) = \frac{1}{4}$$

CANDIDATI $f(1) = \frac{1}{5}$

$$f(2) = \frac{1}{4}$$

$$f(5) = \frac{5}{29}$$

IL VALORE MASSIMO GLOBALE

$$\frac{1}{4} \quad \text{PUNTO STAZIONARIO}$$

IL VALORE MINIMO GLOBALE
IN $[1, 5]$

$$\frac{5}{29}$$

2) RETTA TG

$$y - f(x_0) = f'(x_0)(x - x_0)$$

IN PUNTO MINIMO GLOBALE $(5, \frac{5}{29})$

$\downarrow$ $\downarrow$
 x_0 $f(x_0)$

$$y - \frac{5}{29} = f'(x_0)(x - 5)$$

$$f'(x_0) = f'(5) = \frac{4 - 25}{(25 + 4)^2} = \frac{-21}{(29)^2}$$

$$y - \frac{5}{29} = -\frac{21}{(29)^2}(x - 5)$$

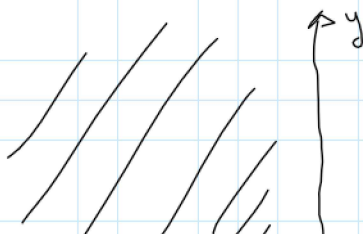
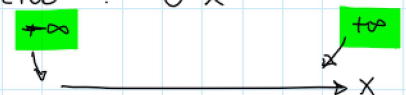
3) DISEGNO IL GRAFICO

$$f(x) = \frac{x}{x^2 + 4}$$

1) DOMINIO $x^2 + 4 \neq 0 \rightarrow$ SEMPRE VERO: $\forall x$

$$x^2 \neq -4$$

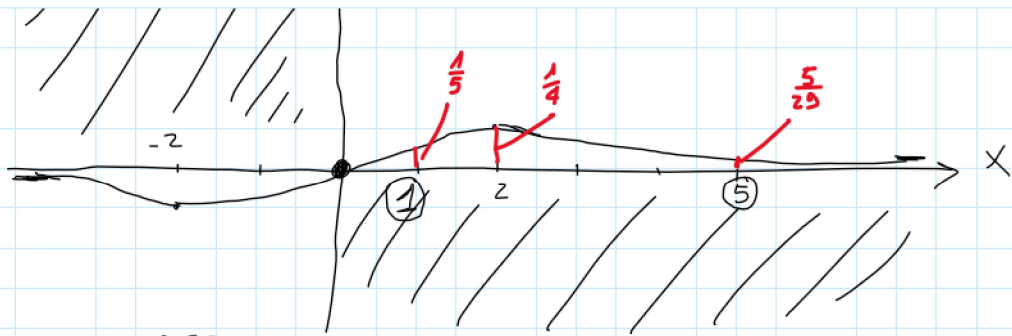
UN QUADRATO E SEMPRE $\neq 0$



$$\frac{1}{5}$$

$$\frac{1}{4}$$

$$\frac{5}{29}$$



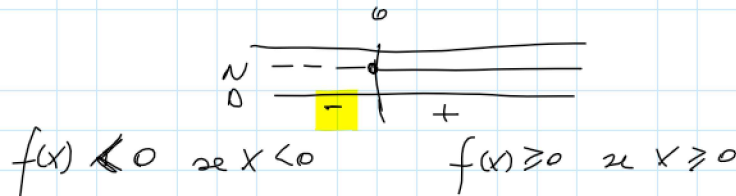
2) INT. ASSI

$$\begin{cases} x=0 \\ y=0 \end{cases} \quad O(0,0) \quad \begin{cases} y=0 \\ \frac{x}{x^2+4}=0 \end{cases} \quad x=0$$

3) STUDIO SEGNO $f(x) > 0$ o ≥ 0

$$\frac{x}{x^2+4} \geq 0 \quad \text{FRATTA} \quad \begin{matrix} N \geq 0 & x \geq 0 \\ D > 0 & x^2+4 > 0 \quad \forall x \end{matrix}$$

SOPRA DI 2 QUANTITÀ POSITIVE



4) ASINTOTI:

VERTICALI: NON CI SONO PERCHÉ $\exists \forall x$
ORIZZONTALI

$$\lim_{x \rightarrow +\infty} \frac{x}{x^2+4} = 0 \quad y=0 \text{ A.O.R.}$$

$$\lim_{x \rightarrow -\infty} \frac{x}{x^2+4} = 0$$

5) MAX e min $f'(x) = 0$ $f'(x) > 0$ ($f'(x) \geq 0$)

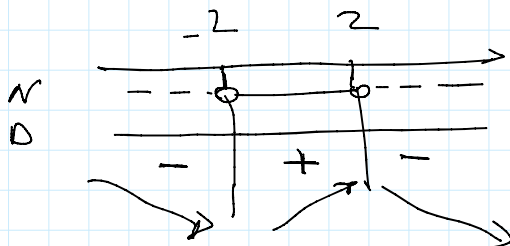
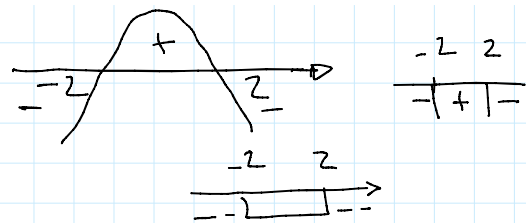
$\Downarrow$
 $x = \pm 2$
MAX, minimo,
FLESSI TG ORIZ

$\Downarrow$
SCOPRO TUTTO

$$\frac{4-x^2}{(x^2+4)^2} > 0 \quad N \geq 0 \quad 4-x^2 > 0 \quad 2^\circ \text{ grado} = 0$$

$$4-x^2 = 0 \quad x^2 = 4 \quad x = \pm 2$$

$$D > 0 \quad (x^2 + 4)^2 > 0 \quad \forall x$$



$x = -2$ minimo locale

$x = 2$ max locale

$$f(-2) = \frac{-2}{8} = -\frac{1}{4}$$

$$f(2) = \frac{1}{4}$$

6) FLESSI $f''(x) \geq 0$

PER VOI

$$f(x) = \frac{x^2 + 4}{x} \quad I [-7, -1]$$

$$f(x) = \frac{x^2 + 1}{x} \quad I [\frac{1}{2}, 3]$$

RIPETERE I PUNTI
1) 2) 3) DELL'ESERCIZIO
PRECEDENTE (12-01-2017)

DATA $f(x) = \left| \frac{x+2}{x-1} \right|$ DISEGNA IL GRAFICO

QUANDO HO IL VALORE ASSOLUTO SE POSSO ME LO
TENGO FINO AL MOMENTO DI DERIVARE.
POI LO DEVO SCIOGLIERE - SE HO PROBLEMI LO SCIOLGO PRIMA

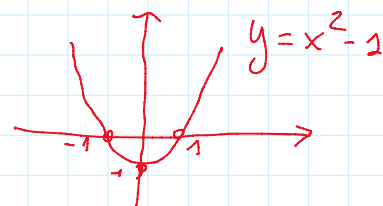
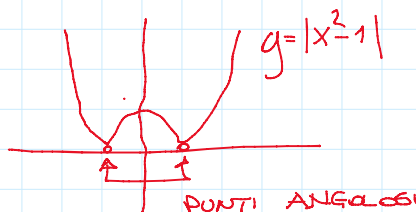
QUESTO È UN ESERCIZIO PARTICOLARE

$y = |f(x)|$ POSSO GRAFICARE $y = f(x)$

E LA PARTE NEGATIVA LA RIPORTO SOPRA L'ASSE X

$$\text{ES } y = |x^2 - 1| \rightarrow y = x^2 - 1$$

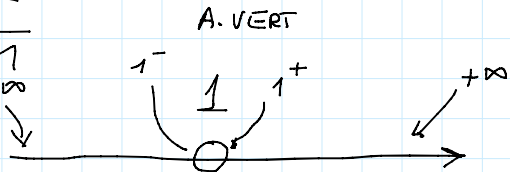
$$y = |x^2 - 1|$$



$$y = \left| \frac{x+2}{x-1} \right| \quad \text{o} \quad f(x) = \frac{x+2}{x-1}$$

GRAFICO $f(x) = \frac{x+2}{x-1}$

1) DOMINIO $x \neq 1$



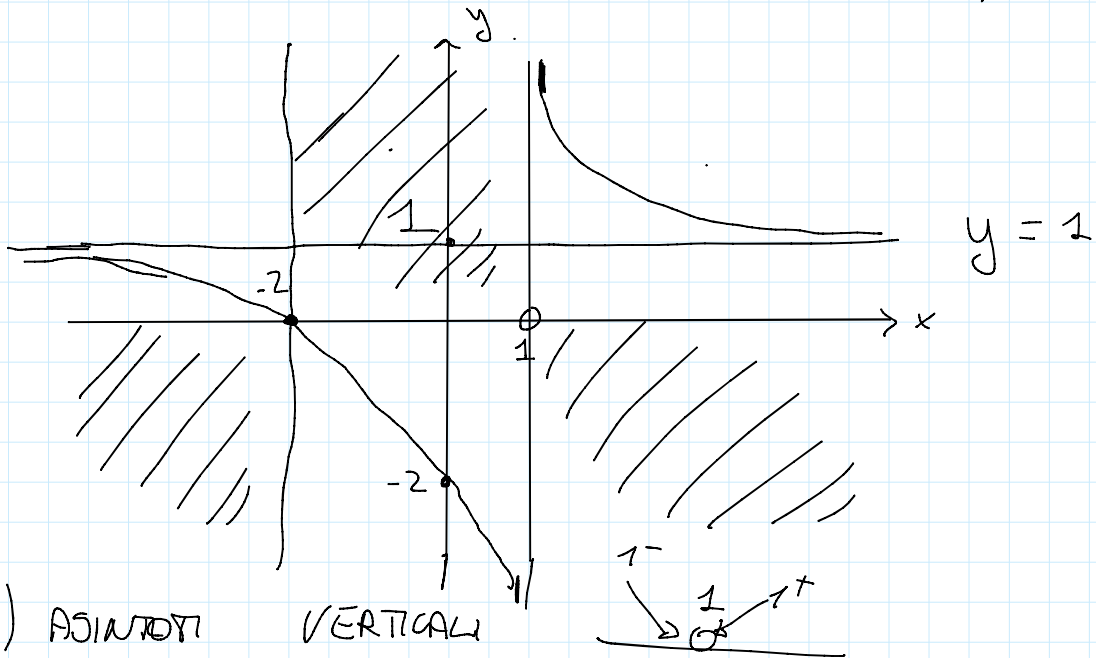
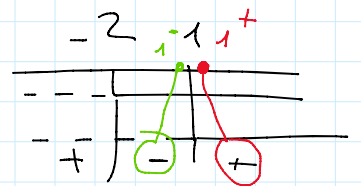
2) INT. ASSI

$$\begin{cases} x=0 \\ y=-2 \end{cases} \quad (0, -2) \quad \begin{cases} y=0 \\ \frac{x+2}{x-1}=0 \end{cases} \quad x=-2 \quad (-2, 0)$$

3) SEGNO

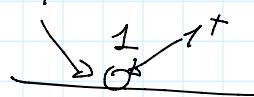
$$\frac{x+2}{x-1} \geq 0$$

$$\begin{matrix} N & x \geq -2 \\ D & x > 1 \end{matrix}$$



4) ASINTOTI

VERTICALI



$$\lim_{x \rightarrow 1^+} \frac{x+2}{x-1} = \frac{3}{0^+} = +\infty$$

DEVO VEDERE SE È 0^+ o 0^- ?

$$\lim_{x \rightarrow 1^-} \frac{x+2}{x-1} = \frac{3}{0^-} = -\infty \quad 0^+ \text{ o } 0^-?$$

$x=1$ A. VERT. (2° SPECIE)

ORIZZONTALI

$$\lim_{x \rightarrow \pm\infty} \frac{\boxed{x}+2}{\boxed{x}-1} = 1 \quad y=1 \text{ A.O.R.}$$

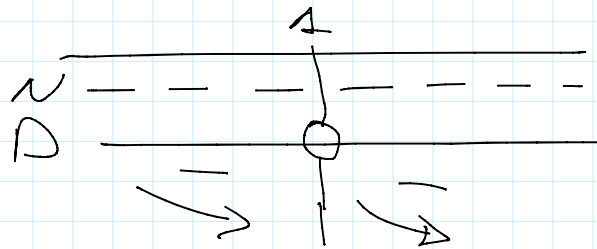
5) MAX e min

$$f(x) = \frac{\boxed{x+2}}{\boxed{x-1}}$$

$$f'(x) = \frac{1(x-1) - (x+2)1}{(x-1)^2} = \frac{x-1-x-2}{(x-1)^2} = \frac{-3}{(x-1)^2}$$

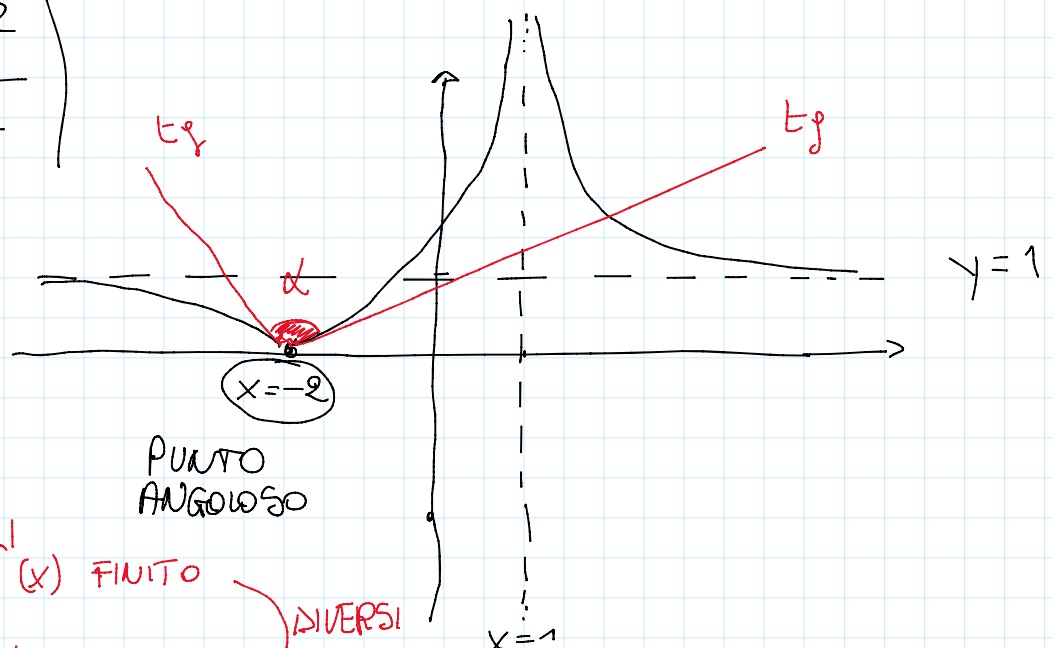
$$f'(x) = 0 \quad - \frac{3}{(x-1)^2} = 0 \quad \text{IMP}$$

$$f'(x) > 0 \quad N - 3 \text{ SETPRE} - \quad 0(x-1)^2 > 0 \quad \forall x \in \mathbb{R}$$



NO MAX
NO MIN

$$y = \left| \frac{x+2}{x-1} \right|$$



$$\lim_{x \rightarrow -2^-} f'(x) \text{ FINITO}$$

$$\lim_{x \rightarrow -2^+} f'(x) \text{ FINITO}$$

DIVERSI

$$f(x) = \left| \frac{x}{x-1} \right|$$

$\lim_{x \rightarrow -2^+} f(x)$ FINITO

PER VOI...

$$f(x) = \left| \frac{x}{x^2 - 1} \right|$$

$$f(x) = \frac{x-1}{4-x^2}$$

DISegnARE

ESERCIZI VOLTA SCORSA

1° ESERCIZIO

→ 1) $a=3, a=13$
 2) $a=8$

$$f(x) = \begin{cases} -2x+8 & x < 1 \\ x+5 & x \geq 1 \end{cases}$$

2° ESERCIZIO

→ 1) $k=-1 \vee k=3$
 2) $k=1$

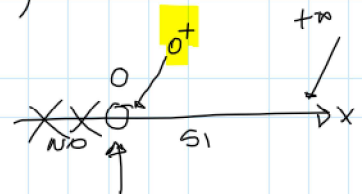
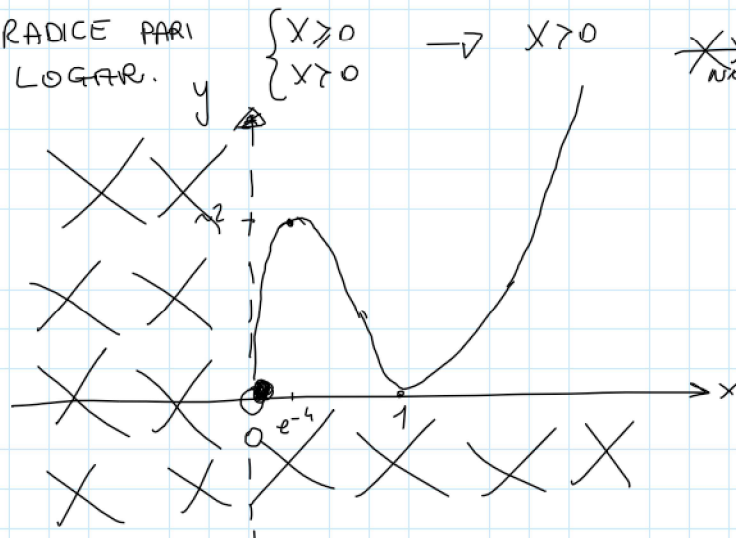
$$f(x) = \begin{cases} e^x + 1 & x < 0 \\ x^2 - 3x + 2 & x \geq 0 \end{cases}$$

$$f(x) = \sqrt{x} \cdot \ln^2 x \quad (\dots 2019)$$

1) DOMINIO

RADICE PARI
 LOGAR.

$$\begin{cases} x \geq 0 \\ x > 0 \end{cases} \rightarrow x > 0$$



INT. ASS.

$x=0$? NON SI PUÒ PERCHÉ $0 \notin D$

$$y=0$$

$$\sqrt{x} \cdot \ln^2 x = 0$$

PER

ZERO

$$\begin{cases} \sqrt{x} = 0 \\ \ln^2 x = 0 \end{cases}$$

$$\sqrt{x} = 0 \rightarrow x = 0? \quad 0 \notin D$$

$$\ln x = 0 \quad x = 1$$

"SEPARO"

$$\begin{cases} \log_b b = c & b = a^c \\ \ln x = 0 & x = e^0 = 1 \end{cases}$$

DA $\sqrt{x} = 0$ HO OTTENUTO $x = 0$

DA $\sqrt{x}=0$ HO OTTENUTO $x=0$

$$\lim_{x \rightarrow 0^+} x = 0 \quad x = e^0 = 1$$

MA $x=0 \notin D$. ALLORA MI DEVO "INSEGUIRE"

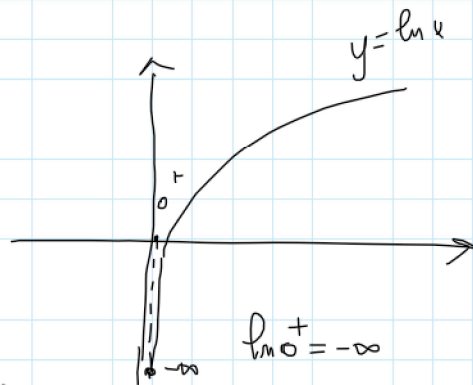
È AVERE CHIARIMENTI COL $\lim_{x \rightarrow 0^+} f(x)$. LUI MI DIRÀ TUTTO

3) SEGNO

$$\sqrt{x} \cdot \ln^2 x \geq 0 \quad \text{SEPARO}$$

$$F_1 \quad \sqrt{x} \geq 0 \quad \forall x \in D \quad (\text{LA RADICE È POSITIVA})$$

$$F_2 \quad \ln^2 x \geq 0 \quad (\ln x)^2 \geq 0 \quad \forall x \in D \quad (\text{È UN } (\)^2)$$



4) ASINTOTI

$$\lim_{x \rightarrow 0^+} \sqrt{x} \cdot \ln^2 x = 0 \cdot (-\infty)^2 = 0 \cdot \infty \quad \text{F.I.}$$

POSIZIONE $x = e^{-t}$

$$x \rightarrow 0 \Rightarrow e^{-t} \rightarrow 0 \Rightarrow \frac{1}{e^t} \rightarrow 0 \Rightarrow e^t \rightarrow \infty \quad t \rightarrow \infty \quad \left(\frac{1}{\infty} = 0\right)$$

$$\lim_{t \rightarrow \infty} \sqrt{e^{-t}} \cdot (\ln e^{-t})^2$$

$$\lim_{t \rightarrow \infty} e^{-\frac{t}{2}} \cdot (-t \cdot \ln e)^2$$

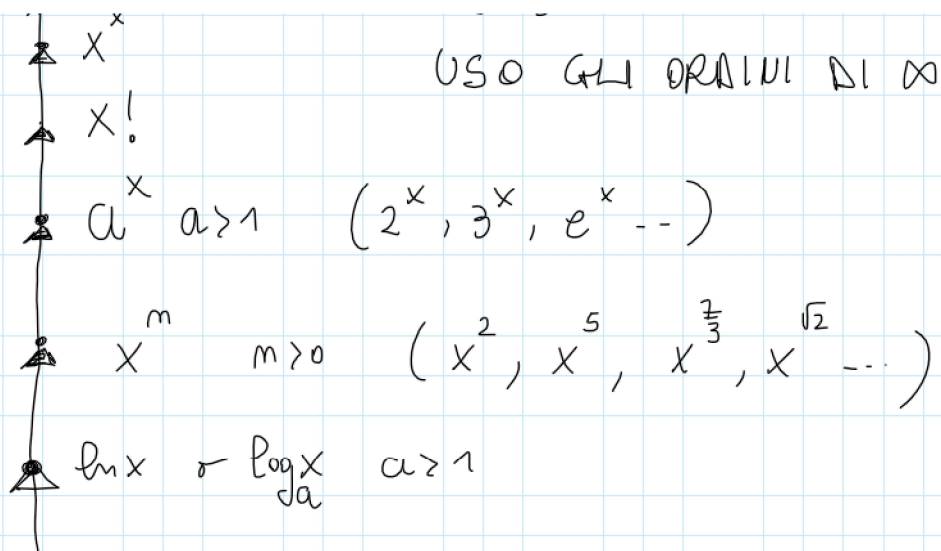
$$\lim_{x \rightarrow \infty} e^{-\frac{1}{2}t} \cdot (-t \cdot 1)^2 = \lim_{t \rightarrow \infty} \frac{t^2 \rightarrow \text{PERDE}}{e^{\frac{1}{2}t} \rightarrow \text{VINCE}} = 0$$

$$\sqrt[m]{a^m} = a^{\frac{m}{m}} = a$$

$$\log_a b^m = m \log_a b \quad \log_a a = 1$$

USO DE L'HOPITAL

USO GLI ORDINI DI ∞



$x \rightarrow +\infty$

L' ∞ CHE PERDE VIENE "DECLASSATO AD UN NUMERO" RISPETTO ALL' ∞ CHE VINCE

ALLORA O "LO RIVALUTIAMO"

$x=0$ NON È AS. VERTICALE

ORIZZ.

$$\lim_{x \rightarrow +\infty} \sqrt{x} \cdot (\ln x)^2 = \infty \quad \nabla \text{ ORIZ}$$

NON CERCO NEANCHE OBLIQUO

5) MAX / min

$f(x) = \sqrt{x} \cdot (\ln x)^2$ È UNA $(f(x))^n$
 DERIVATA $n(f(x))^{n-1} \cdot f'(x)$

$$f'(x) = \frac{1}{2\sqrt{x}} \cdot (\ln x)^2 + \sqrt{x} \cdot 2(\ln x)^{2-1} \cdot \frac{1}{x}$$

$$= \ln x \left[\frac{1}{2\sqrt{x}} \cdot \ln x + \frac{2\sqrt{x}}{x} \right]$$

$$= \ln x \left[\frac{x \ln x + 4 (\cancel{\sqrt{x}})^2}{2x \sqrt{x}} \right]$$

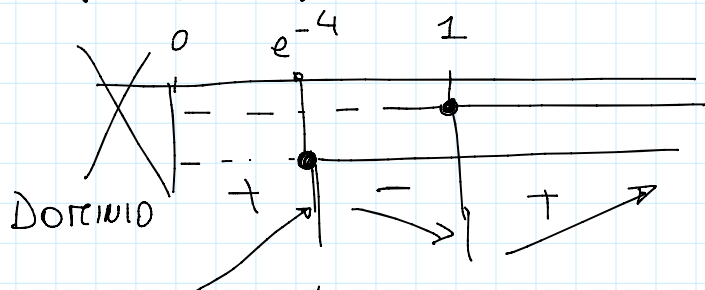
$$= \ln x \left[\frac{x \ln x + 4x}{2x \sqrt{x}} \right]$$

$$= \ln x \left[\frac{x(\ln x + 4)}{2x\sqrt{x}} \right] \geq 0$$

$$\ln x \geq 0 \quad x \geq e^0 \quad x \geq 1$$

$$\ln x + 4 \geq 0 \quad \ln x \geq -4 \quad x \geq e^{-4}$$

$$\sqrt{x} \geq 0 \quad \forall x \in D$$



$$x = e^{-4} \text{ max locale}$$

$$x = 1 \text{ min locale}$$

$$f(1) = 0 \quad \text{GIÀ FATTO} \quad (1, 0)$$

$$f(e^{-4}) = \sqrt[2]{e^{-4}} (\ln e^{-4})^2 = e^{-\frac{4}{2}} (-4 \ln e)^2 =$$

$$= e^{-2} \cdot (-4 \cdot 1)^2 = \frac{16}{e^2} \approx 2.2$$

PER VOI...

$$f(x) = x^3 \cdot \ln x \quad f(x) = |x| e^{-x+1}$$

$$f(x) = \sqrt{x+1} |\ln(x+1)| \quad f(x) = e^{\frac{2x+1}{x-2}}$$

$$y = \sqrt{x+1} |\ln(x+1)|$$

$$1) \quad \begin{array}{l} x+1 \geq 0 \\ x+1 > 0 \end{array} \quad \dots$$

$$2) \quad \begin{cases} x=0 \\ y = \sqrt{1} \cdot |\ln(1)| = 1 \cdot |\ln 1| = 1 \cdot 0 = 0 \end{cases}$$

$$\text{sego} \quad \sqrt{x+1} \cdot |\ln(x+1)| \geq 0$$

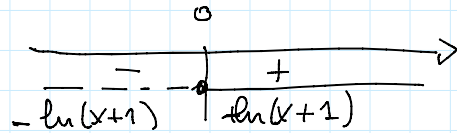
$$\sqrt{x+1} \geq 0 \quad \forall x \in D$$

$$\sqrt{x+1} \geq 0 \quad \forall x \in \mathbb{D} \quad \sqrt{\quad}$$

$$|-\ln(x+1)| \geq 0 \quad \forall x \in \mathbb{D} \quad | \quad |$$

COME LO SCIOLGO $\rightarrow$ "ASINTOTI" DERIVATA.

$$\text{argomento} \geq 0 \quad \ln(x+1) \geq 0 \quad (x+1) \geq 1 \quad x \geq 0$$



$$f(x) = \begin{cases} \sqrt{x+1} \cdot (-\ln(x+1)) & \text{se } x < 0 \\ \sqrt{x+1} \cdot (\ln(x+1)) & \text{se } x \geq 0 \end{cases} \quad \bullet \bullet \bullet \bullet$$