

APPELLO 13.02.2024

$$\int \frac{x + \arctan x}{1+x^2} dx$$

$$\int \frac{x}{1+x^2} dx + \int \frac{\arctan x}{1+x^2} dx$$

$$1+x^2 = t$$

DERIVO

$$2x dx = dt$$

$$x dx = \frac{1}{2} dt$$

$$\arctan x = t$$

DERIVO

$$\frac{1}{1+x^2} dx = dt$$

$$\int t dt = \frac{1}{2} t^2 = \frac{1}{2} \arctan^2 x$$

$$\int \frac{1}{t} \cdot \frac{1}{2} dt = \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \ln|t| = \frac{1}{2} \ln|1+x^2| = \frac{1}{2} \ln(1+x^2)$$

$$\int \frac{x + \arctan x}{1+x^2} dx = \frac{1}{2} \ln(1+x^2) + \frac{1}{2} \arctan^2 x + c$$

$$\int \frac{x + 1 + \ln x}{x} dx$$

$$\int_1^e \frac{\sqrt{\ln x}}{x} dx$$

$$\int \frac{x^1}{x} dx + \int \frac{1}{x} dx + \int \frac{\ln x}{x} dx$$

$$1x + \ln|x|$$

$$\ln x = t$$

$$\frac{1}{x} dx = dt$$

$$\int t dt = \frac{1}{2} t^2$$

$$\frac{1}{2} \ln^2 x$$

$$= x + \ln|x| + \frac{1}{2} \ln^2 x + c$$

$$\ln x = t$$

$$\frac{1}{x} dx = dt$$

ESTREMI

$$x=1 \quad \ln 1 = t$$

$$t=0$$

$$x=e \quad \ln e = t$$

$$t=1$$

$$\int_0^1 \sqrt{t} dt$$

$$\int_0^1 t^{\frac{1}{2}} dt = \left| \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right|_0^1$$

$$= x + \ln|x| + \frac{1}{2}\ln^2 x + c$$

$$\int_0^1 t^{\frac{1}{2}} dt = \left| \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right|_0^1$$

$$\left| t^{\frac{3}{2}} \cdot \frac{2}{3} \right|_0^1 = \frac{2}{3} - 0 = \frac{2}{3}$$

$$\int \frac{e^{2x} + e^x}{e^{2x} + 7e^x + 10} dx$$

$$e^x = t$$

DERIV $e^x dx = 1 dt$

$$\int \frac{e^x (e^x + 1)}{t^2 + 7t + 10} dx$$

$$\int \frac{t+1}{t^2+7t+10} dt$$

$$\frac{1t+1}{t^2+7t+10} = \frac{A}{t+5} + \frac{B}{t+2} = \frac{At+2A+Bt+5B}{(t+5)(t+2)}$$

$$= \frac{(A+B)t + 2A+5B}{t^2+7t+10} \quad \begin{cases} A+B=1 \\ 2A+5B=1 \end{cases}$$

$$\begin{array}{rcl} -2 & \begin{cases} -2A - 2B = -2 \\ 2A + 5B = 1 \end{cases} & \\ \hline // & 3B = -1 & B = -\frac{1}{3} \end{array}$$

$$\begin{array}{rcl} -5 & \begin{cases} -5A - 5B = -5 \\ 2A + 5B = 1 \end{cases} & \\ \hline -3A & // & = -4 \\ & & A = \frac{4}{3} \end{array}$$

$$\int \frac{t+1}{t^2+7t+10} dt = \frac{4}{3} \int \frac{1}{t+5} dt - \frac{1}{3} \int \frac{1}{t+2} dt$$

$$\frac{4}{3} \ln|t+5| - \frac{1}{3} \ln|t+2| + c \quad e^x = t$$

$$\frac{4}{3} \ln(e^x+5) - \frac{1}{3} \ln(e^x+2) + c.$$

$$\frac{4}{3} \ln(e^x + 5) - \frac{1}{3} \ln(e^x + 2) + c.$$