

$$\sum_{n=1}^{+\infty} \frac{\ln n}{n^3}$$

$$\lim_{n \rightarrow +\infty} \frac{\ln n}{n^3} = 0 \quad \text{PER L'ORDINE DI } \infty$$

VINCE

$$a_n = \frac{(\ln n)^1}{n^3} = \frac{1}{n^3 \cdot (\ln n)^{-1}}$$

$$\sum_{n=1}^{+\infty} \frac{\ln n}{n^3} = \sum_{n=1}^{+\infty} \frac{1}{n^3 (\ln n)^{-1}}$$

$\alpha = 3$   
 $\beta = -1$   
 $\alpha > 1$  CONVERGE  $\forall \beta$

$$\sum_{n=1}^{+\infty} n \ln \left( 1 + \frac{1}{n^3} \right)$$

$$\lim_{n \rightarrow +\infty} n \cdot \ln \left( 1 + \frac{1}{n^3} \right) \sim \lim_{n \rightarrow +\infty} \cancel{n} \cdot \frac{1}{n^{3 \rightarrow 2}} = 0$$

$\downarrow$   
 $\infty \cdot \ln 1$   
 $\infty \cdot 0$  F.I.

$$a_n = n \cdot \ln \left( 1 + \frac{1}{n^3} \right) \sim \frac{1}{n^2}$$

$$\sum_{n=0}^{+\infty} n \ln \left( 1 + \frac{1}{n^3} \right) \sim \sum_{n=0}^{+\infty} \frac{1}{n^2} \quad \alpha > 1 \text{ conv.}$$

$$\sum_{n=1}^{+\infty} \sqrt{n} \cdot \left( \arctan \left( \frac{1}{n^3} \right) \right)$$

$$\lim_{n \rightarrow +\infty} \sqrt{n} \cdot \left( \arctan\left(\frac{1}{n^3}\right) \right)$$

$\downarrow$   
 $\infty$

$\downarrow$   
 $\arctan 0$   
 $\downarrow$   
 $0$

$\infty \cdot 0$  F.I.

$$\begin{aligned} \lim_{n \rightarrow +\infty} \sqrt{n} \cdot \left( \arctan\left(\frac{1}{n^3}\right) \right) &\sim \lim_{n \rightarrow +\infty} \sqrt{n} \cdot \frac{1}{n^3} \\ &= \lim_{n \rightarrow +\infty} n^{\frac{1}{2}} \cdot \frac{1}{n^{3-\frac{1}{2}}} = \lim_{n \rightarrow +\infty} \frac{1}{n^{\frac{5}{2}}} = 0 \end{aligned}$$

CONFRONTO ASINTOTICO

$$\sum_{n=1}^{\infty} a_n \sim \sum_{n=1}^{+\infty} \frac{1}{n^{5/2}} \quad \alpha > 1 \text{ CONV.}$$

$\neq$

$$\sum_{n=1}^{+\infty} n^3 \left( \cos\left(\frac{1}{n}\right) - 1 \right)^2$$

$$\begin{aligned} \lim_{n \rightarrow +\infty} n^3 \left( - \left( 1 - \cos\left(\frac{1}{n}\right) \right) \right)^2 &\sim \lim_{n \rightarrow +\infty} n^3 \left( - \frac{1}{2} \left( \frac{1}{n} \right)^2 \right)^2 \\ &= \lim_{n \rightarrow +\infty} n^3 \cdot \frac{1}{4} \cdot \frac{1}{n^4} = \lim_{n \rightarrow +\infty} \frac{1}{4} \cdot \frac{1}{n} = 0 \end{aligned}$$

$$a_n = n^3 \left( \cos\left(\frac{1}{n}\right) - 1 \right)^2 \sim \frac{1}{4} \cdot \frac{1}{n}$$

$$\sum_{n=1}^{+\infty} a_n \sim \frac{1}{4} \sum_{n=1}^{+\infty} \frac{1}{n} \quad \alpha = 1 \text{ DIVERGE}$$

$$\sum_{n=2}^{+\infty} \frac{2^n + \ln^3 n}{n!}$$

CONDIZIONE NECESSARIA

$$\lim_{n \rightarrow +\infty} \frac{2^n + \ln^3 n}{n!} \sim \lim_{n \rightarrow +\infty} \frac{2^n}{n!} = 0 \text{ PER L'ORDINE DI } \infty$$

CRITERIO RAPPORTO

$$a_n = \frac{2^n}{n!} \quad a_{(n+1)} = \frac{2^{(n+1)}}{(n+1)!}$$

$$\lim_{n \rightarrow +\infty} \frac{a_{(n+1)}}{a_n} = \lim_{n \rightarrow +\infty} \frac{2^{(n+1)}}{(n+1)!} \cdot \frac{n!}{2^n}$$

$$\lim_{n \rightarrow +\infty} \frac{2 \cdot 2^n}{n! (n+1)} \cdot \frac{n!}{2^n} = 0 < 1 \text{ CONVERGE.}$$

$$\sum_{n=1}^{+\infty} \frac{3^n}{n^5 \cdot 2^{2n}}$$

~ NO PERCHÉ  
NON CI SONO + e/o -

$$\lim_{n \rightarrow +\infty} \frac{3^n}{n^5 \cdot 2^{2n}} = \lim_{n \rightarrow +\infty} \frac{3^n}{n^5 \cdot 4^n} = \lim_{n \rightarrow +\infty} \frac{1}{n^5} \cdot \left(\frac{3}{4}\right)^n = 0 \cdot \left(\frac{3}{4}\right)^{+\infty} = 0$$

CRITERIO RADICE

$$\lim_{n \rightarrow +\infty} \sqrt[n]{\frac{3^n}{n^5 \cdot 4^n}} = \lim_{n \rightarrow +\infty} \frac{\sqrt[n]{3^n}}{\sqrt[n]{n^5 \cdot 4^n}}$$

$$\lim_{n \rightarrow +\infty} \sqrt[n]{\frac{3}{m^5 \cdot 4^m}} = \lim_{n \rightarrow +\infty} \frac{3}{\sqrt[n]{m^5} \cdot \sqrt[n]{4^m}}$$

$$\lim_{n \rightarrow +\infty} \frac{3}{\sqrt[n]{m^5} \cdot 4} = \lim_{n \rightarrow +\infty} \frac{3}{m^{\frac{5}{n}} \cdot 4}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

uso  $a_n = e$

LA USO QUANDO  
M (MA ANCHE X)  
È PRESENTE SIA  
NELLA BASE, SIA  
NELL'ESPOONENTE

$$\lim_{n \rightarrow +\infty} \frac{3}{e^{\ln m^{\frac{5}{n}}} \cdot 4}$$

$$\lim_{n \rightarrow +\infty} \frac{3}{e^{\frac{5}{n} \cdot \ln m} \cdot 4} = \lim_{n \rightarrow +\infty} \frac{3}{e^{\frac{5 \ln m}{n}} \cdot 4}$$

$$\lim_{n \rightarrow +\infty} \frac{3}{e^0 \cdot 4} = \frac{3}{4} < 1 \text{ CONVERGE}$$

o

$$\sum_{n=2}^{+\infty} \frac{\sin(e \ln n)}{n^2 \ln n}$$

CRITERIO CONFRONTO: È PIÙ CORRETTO FARE UNA  
DOPPIA MAGGIORAZIONE

$$\frac{\sin(e \ln n)}{n^2 \ln n} \leq \frac{|\sin(e \ln n)|}{n^2 \ln n} \leq \frac{1}{n^2 \ln n}$$

CONVERGE ← CONVERGE ASSOLUTAMENTE ←  $\alpha > 1$  CONVERGE