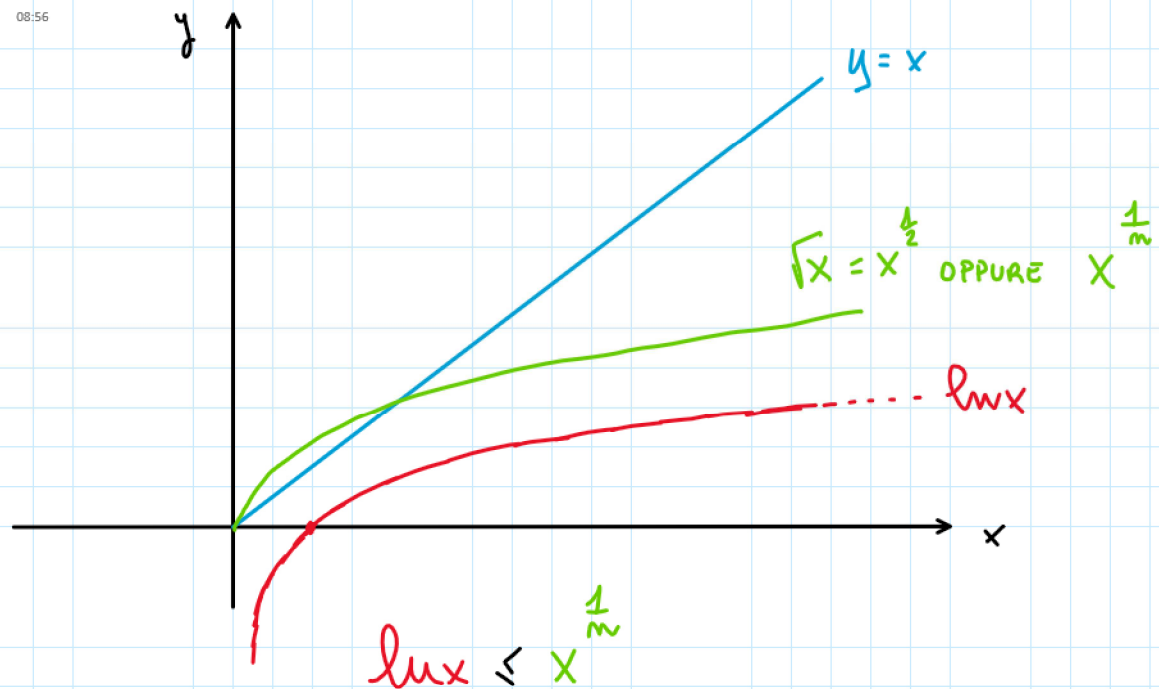




$$\sum_{n=1}^{+\infty} \frac{\ln n}{n^3} = \sum_{n=1}^{+\infty} \frac{1}{n^3 (\ln n)^{-1}}$$

$\alpha = 3$
 $\alpha > 1$ CONVERGE $\forall \beta$

$$\ln n < n$$

$$\frac{\ln n}{n^3} < \frac{n}{n^3}$$

$$\frac{\ln n}{n^3} < \frac{n}{n^3} = \frac{1}{n^2}$$

$$\sum_{n=1}^{+\infty} \frac{\ln n}{n^3} < \sum_{n=1}^{+\infty} \frac{1}{n^2}$$

$\hookrightarrow \alpha > 1$ CONVERGES $\Rightarrow \sum_{n=1}^{+\infty} \frac{\ln n}{n^3}$ CONVERGES

$$\sum_{n=2}^{+\infty} \frac{\text{see}(\ln(n))}{n^2 \ln n}$$

$$a_n = \frac{\text{see}(\ln(n))}{n^2 \ln n} \leq \frac{1}{n^2 \ln n}$$

$\alpha = 2$ $\beta = 1$
 SO CONVERGES

$$\boxed{\frac{1}{n^2 \ln n} \leq \frac{1}{n^\alpha} \quad \alpha > 1} \quad ? \alpha$$

$$n^2 \ln n \geq n^\alpha \quad \ln n \geq \frac{n^\alpha}{n^2}$$

$$\ln n \geq n^{\alpha-2} \quad \alpha = 2$$

$$\ln n \geq 1$$



$$n \rightarrow +\infty \quad \ln n \geq 1$$

MAGGIORAZIONE

$$\frac{1}{n^2 \ln n}$$

$$\text{SO CHE } \ln n \geq 1 \quad n \rightarrow +\infty$$

INVERTO

$$\frac{1}{\ln n} \leq 1$$

$$\frac{1}{n^2 \ln n} = \frac{1}{n^2} \cdot \frac{1}{\ln n} \leq \frac{1}{n^2} \cdot 1$$

$$\sum_{n=2}^{+\infty} \frac{1}{n^2 \ln n} \leq \sum_{n=2}^{+\infty} \frac{1}{n^2} \quad \alpha > 1 \quad \text{CONVERGE}$$

✓

$$\sum_{n=1}^{\infty} (-1)^n \ln \left(\frac{n + \sqrt{n}}{n} \right) \quad \text{:= an}$$

$$1) \lim_{n \rightarrow +\infty} \ln \left(\frac{n + \sqrt{n}}{n} \right) \sim \lim_{n \rightarrow +\infty} \ln \left(\frac{n}{n} \right) = \ln 1 = 0$$

$$\lim_{n \rightarrow +\infty} a_n = 0 \quad \text{OK}$$

2) a_n DECRESCA

$$e^{a_n} \quad \ln a_n \quad \arctan a_n \quad \tan a_n$$

crescono se a_n cresce
decrescono se a_n decresce

$$\ln\left(\frac{n+\sqrt{n}}{n}\right) \quad ? \quad \nearrow \quad ? \quad \searrow$$

$$\text{LAVORO SU } \frac{n+\sqrt{n}}{n} = \frac{n}{n} + \frac{\sqrt{n}}{n}$$

$$= 1 + \frac{n^{\frac{1}{2}}}{n} = 1 + \frac{1}{n^{1-\frac{1}{2}}}$$

$$= 1 + \frac{1}{n^{\frac{1}{2}}} = \boxed{1 + \frac{1}{\sqrt{n}}}$$

DECRESCE

1° modo

1 L'ANDAMENTO ME LO
DA' $\frac{1}{\sqrt{n}}$

so che $\sqrt{n}$ cresce
 $\Rightarrow \frac{1}{\sqrt{n}}$ DECRESCE

2° modo

$$\cancel{\ln\left(1 + \frac{1}{\sqrt{n}}\right)} > \cancel{\ln\left(1 + \frac{1}{\sqrt{n+1}}\right)}$$

$$\cancel{1 + \frac{1}{\sqrt{n}}} > \cancel{1 + \frac{1}{\sqrt{n+1}}}$$

$$\sqrt{n} < \sqrt{n+1}$$

$$n < n+1 \quad \text{VERO } \forall n$$

RIASSUNTO

$$1) \lim_{n \rightarrow +\infty} a_n = 0 \quad 2) a_n \text{ DECRESCE}$$

LA SERIE CONVERGE PER LEIBNIZ

SE VOGLIO VEDERE LA CONVERGENZA
ASSOLUTA

$$|(-1)^n \ln(n+\sqrt{n})| = \ln(n+\sqrt{n})$$

$$\left| (-1)^n \ln \left(\frac{n+\sqrt{n}}{n} \right) \right| = \ln \left(\frac{n+\sqrt{n}}{n} \right)$$

$$\sum_{n=1}^{+\infty} \ln \left(\frac{n+\sqrt{n}}{n} \right)$$

$$\ln \left(\frac{n+\sqrt{n}}{n} \right) = \ln \left(1 + \frac{1}{\sqrt{n}} \right) \sim \frac{1}{\sqrt{n}}$$

$$= \frac{1}{n^{1/2}} \quad \alpha = \frac{1}{2} \quad \text{DIVERGE}$$

$$\sum_{n=1}^{+\infty} \left| (-1)^n \ln \left(\frac{n+\sqrt{n}}{n} \right) \right| \quad \text{DIVERGE}$$

$$\sum_{n=1}^{\infty} (-1)^n \boxed{\frac{|\sin n|}{n\sqrt{n}+1}} \quad \text{:= } a_n$$

$$\lim_{n \rightarrow +\infty} \frac{|\sin n|}{n\sqrt{n}+1} \leq \lim_{n \rightarrow +\infty} \frac{1}{n\sqrt{n}+1} = 0$$

$$1) \lim_{n \rightarrow +\infty} a_n = 0$$

$$\frac{|\sin n|}{n\sqrt{n}+1} \text{ decreases?} \quad \text{NON SI PUÒ STUDIARE}$$

$$\sum_{n=1}^{+\infty} \frac{\sin(n)}{n\sqrt{n}+2}$$

$$\lim_{n \rightarrow +\infty} \frac{\sin n}{n\sqrt{n}+2} = 0 \quad \text{LA SERIE POTREBBE CONVERGERE}$$

CONFRONTO + CONFRONTO ASINTOTICO

$$\frac{\sin(n)}{n\sqrt{n}+1} \leq \frac{|\sin(n)|}{n\sqrt{n}+1} \leq \frac{1}{n\sqrt{n}+1} \sim \frac{1}{n\sqrt{n}} = \frac{1}{n \cdot n^{1/2}} = \frac{1}{n^{3/2}}$$

$$\alpha = \frac{3}{2} \quad \alpha > 1 \quad \text{CONVERGENTE}$$

STUDIARE LA MONOTONIA E DETERMINARE

SUP; INF; SPECIFICANDO SE SONO MAX O MIN

$$a_n = \ln\left(2 + \frac{1}{n}\right) \quad n \in \mathbb{N} - \{0\}$$

$$a_1 = \ln(2+1) = \ln 3$$

$$a_2 = \ln\left(2 + \frac{1}{2}\right) = \ln \frac{5}{2}$$

$$a_3 = \ln\left(2 + \frac{1}{3}\right) = \ln\left(\frac{7}{3}\right)$$

DIMOSTRIAMO CHE a_n DECRESCERE $a_n > a_{(n+1)}$

$$\cancel{\ln}\left(2 + \frac{1}{n}\right) > \cancel{\ln}\left(2 + \frac{1}{(n+1)}\right)$$

$$\cancel{2} + \frac{1}{n} > \cancel{2} + \frac{1}{(n+1)} \quad \text{INVERTO}$$

$$\cancel{n} < \cancel{n} + 1 \quad 0 < 1 \quad \forall n$$

ALLORA a_n DECRESCERE

$$a_1 = \ln 3$$

$\rightarrow \rightarrow$

VALORE PIÙ GRANDE È a_n
MAX

$$\lim_{n \rightarrow \infty} \ln\left(2 + \frac{1}{n}\right) = \ln 2$$

VALORE PIÙ PICCOLO È a_n
INF.



INF.

ψ

$$a_n = e^{\pi + \frac{1}{n}} \quad n \in \mathbb{N} - \{0\}$$

$$a_1 = e^{\pi + 1}$$

$$a_2 = e^{\pi + \frac{1}{2}}$$

DECRESCÈ?

$$e^{\pi + \frac{1}{n}} > e^{\pi + \frac{1}{n+1}}$$

PASSANDO AGLI ESPONENTI

$$\cancel{\pi} + \frac{1}{n} > \cancel{\pi} + \frac{1}{n+1}$$

a_n DECRESCÈ

$$a_1 = e^{\pi + 1} \quad \text{MAX} \quad n < n+1 \quad \forall n$$

$$\rightarrow \lim_{n \rightarrow +\infty} e^{\pi + \frac{1}{n}} = e^{\pi} \quad \text{INF}$$

d

$$a_n = (-1)^n \frac{2}{n^2 - 1} \quad n \in \mathbb{N} - \{1\}$$

$$n \text{ PARI} \quad (-1)^n = 1$$

$$a_n = \frac{2}{n^2 - 1} \quad n = 2p$$

$$a_{2p} = \frac{2}{4p^2 - 1} \quad \text{DECRESCÈ}$$

1° MODO

$$a_{2p} > a_{2p+2}$$

$$\frac{\cancel{2}}{4p^2 - 1} > \frac{\cancel{2}}{(2p+2)^2 - 1}$$

$$4p^2 - \cancel{1} < (2p+2)^2 - \cancel{1}$$

$$n \text{ DISPARI} \quad (-1)^n = -1$$

$$a_n = \frac{2}{n^2 - 1} \quad n = 2p+1$$

$$a_{2p+1} = -\frac{2}{(2p+1)^2 - 1} \quad \begin{matrix} p \neq 0 \\ \text{così} \\ n \neq 1 \end{matrix}$$

$$a_3 = -\frac{2}{8} = -\frac{1}{4}$$

$$a_5 = -\frac{2}{24} = -\frac{1}{12}$$

$$a_{\dots} < a_{\dots}$$

$$4p^2 - 1 < (2p+2)^2 - 1$$

$$4p^2 < 4p^2 + 8p + 4$$

$$0 < 8p + 4 \quad \forall p \in \mathbb{N}$$

$$a_0 = -2$$

$$a_2 = \frac{2}{3}$$

$$a_4 = \frac{2}{15}$$

$$a_0 = -2 \rightarrow a_2 = \frac{2}{3}$$

$$a_2 = \frac{2}{3} \rightarrow a_4 = \frac{2}{15}$$

$$\lim_{p \rightarrow +\infty} \frac{2}{4p^2 - 1} = 0$$

CRESCERE

$$a_3 = -\frac{1}{4} \rightarrow \lim_{p \rightarrow +\infty} \frac{2}{(2p+1)^2 - 1} = 0$$

$$a_{(2p+1)} < a_{(2p+3)}$$

$$-\frac{2}{(2p+1)^2 - 1} < -\frac{2}{(2p+3)^2 - 1}$$

CAMBIO SEGNO E VERSO È DIVISO PER 2

$$\frac{1}{(2p+1)^2 - 1} > \frac{1}{(2p+3)^2 - 1}$$

INVERTO È CAMBIO VERSO

$$(2p+1)^2 - 1 < (2p+3)^2 - 1$$

$$4p^2 + 4p + 1 < 4p^2 + 12p + 9$$

$$0 < 8p + 8 \quad \forall p$$

RIEPILOGO

m PARI

$$a_0 = -2 \text{ (min)} \rightarrow a_2 = \frac{2}{3} \text{ (MAX)} \rightarrow 0$$

$$\text{MAX} = \frac{2}{3} \quad \text{min} = -2$$

φ

m DISPARI

$$a_3 = -\frac{1}{4} \rightarrow 0$$

$$a_m = \frac{m^2 + m}{m^3 + 1}$$

$$m \in \mathbb{N}$$

$$a_m = \frac{m(m+1)}{(m+1)(m^2 - m + 1)}$$

$$1^3 \cdot 2^3$$

$$(1 \cdot 2) \cdot (1^2 \cdot 2^2)$$

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$

SOMMA DI
2 CUBI

FALSO QUADRATO
~~2AB~~
È IRRIDUCIBILE

$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$

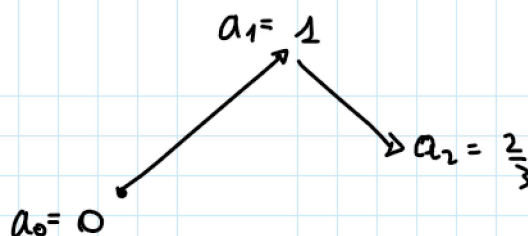
$$a_m = \frac{m}{m^2 - m + 1}$$

$$m=0 \quad a_0 = 0$$

$$m=1 \quad a_1 = 1$$

$$m=2 \quad \frac{2}{4-2+1} = \frac{2}{3}$$

$$m=3 \quad \frac{3}{9-3+1} = \frac{3}{7}$$



$$a_m > a_{m+1} \quad \frac{m}{m^2 - m + 1} > \frac{m+1}{(m+1)^2 - (m+1) + 1}$$

$$\frac{m}{m^2 - m + 1} > \frac{m+1}{m^2 + 2m + 1 - m - 1 + 1}$$

$$\frac{m}{\underbrace{m^2 - m + 1}_{\text{POSITIVI}}} > \frac{m+1}{\underbrace{m^2 + m + 1}_{\text{POSITIVI}}}$$

$$m(m^2 + m + 1) > (m+1)(m^2 - m + 1)$$

$$\cancel{m^3} + m^2 + m > \cancel{m^3} - \cancel{m^2} + m + \cancel{m^2} - m + 1$$

$$\textcircled{+} m^2 + m - 1 > 0$$

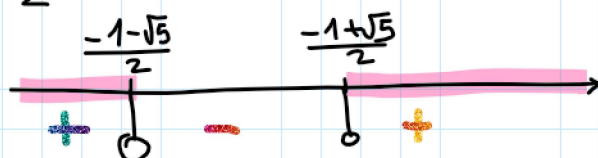
$$m^2 + m - 1 = 0$$

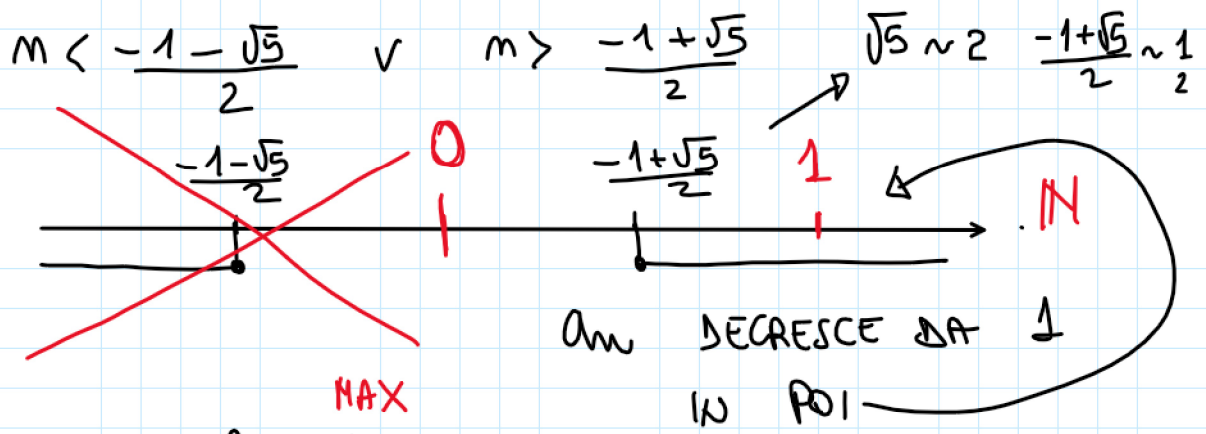
$$\Delta = b^2 - 4ac = 1 - 4(-1) = 5$$

$$m_{1,2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$m_1 = \frac{-1 - \sqrt{5}}{2} \quad \text{NEGATIVO}$$

$$m_2 = \frac{-1 + \sqrt{5}}{2} \quad \text{POSITIVO}$$





$a_0 = 0$ **MIN**
 $a_1 = 1$ **MAX**
 CRESCERE

$a_2 = \frac{2}{3}$
 $a_3 = \frac{3}{7}$
 DECRESCERE

$\lim_{n \rightarrow \infty} a_n = 0$

$\emptyset$

$\lim_{x \rightarrow 1} \frac{3x-3}{2x+1} = 0$ VERIFICA

$\forall \varepsilon > 0 \exists \delta_\varepsilon$ tale che $|x - x_0| < \delta_\varepsilon \Rightarrow |f(x) - l| < \varepsilon$
 con $x \neq x_0$

$\left| \frac{3x-3}{2x+1} - 0 \right| < \varepsilon$

$\left| \frac{3x-3}{2x+1} \right| < \varepsilon$

$x \rightarrow 1$ PER CUI $x > 0$

$|2x+1| > |0+1|$

$|2x+1| > 1$

$\frac{1}{|2x+1|} < 1$

$\left| \frac{3x-3}{2x+1} \right| = |3x-3| \cdot \frac{1}{|2x+1|} < |3x-3| \cdot 1 < \varepsilon$

$\delta_\varepsilon \quad |3x-3| < \varepsilon$

ANCHE

$$\left| \frac{2x-3}{2x+1} \right| \tilde{\epsilon} < \epsilon$$

LAVORO SU $|3x-3| < \epsilon$

$$-\epsilon < 3x-3 < \epsilon$$

$$-\epsilon < 3(x-1) < \epsilon \quad (: 3)$$

$$-\frac{\epsilon}{3} < x-1 < \frac{\epsilon}{3}$$

$$-\frac{\epsilon}{3} + 1 < x < \frac{\epsilon}{3} + 1$$

$$\delta_\epsilon = \frac{\epsilon}{3}$$

