

$$1) \sum_{n=0}^{+\infty} \frac{3^n + n^2}{2^n + n!}$$

È SAGGIO ALL'INIZIO TENTARE DI USARE L'ASINTOTICITÀ PER "SCREENARE"

$a_n$ , COE' ELIMINARE TERMINI CHE VANNO  $\rightarrow +\infty$  PIÙ LENTAMENTE

**CRITERIO CONFRONTO ASINTOTICO: ORALE**

$$a_n = \frac{3^n + n^2}{2^n + n!} \sim b_n = \frac{3^n}{n!}$$

IN QUESTA SITUAZIONE PROVATO CRITERIO RAPPORTO

$$b_{(n+1)} = \frac{3^{(n+1)}}{(n+1)!} \quad \lim_{n \rightarrow +\infty} \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} =$$

$$\lim_{n \rightarrow +\infty} \frac{\cancel{3^n} \cdot 3}{n! (n+1)} \cdot \frac{n!}{\cancel{3^n}} = 0 < 1 \quad \sum b_n \text{ CONVERGE} \Rightarrow \sum a_n \text{ CONV.}$$

$$2) \sum_{n=0}^{+\infty} \frac{2 + n!}{(n+3)!}$$

$$a_n = \frac{2 + n!}{(n+3)!} \sim b_n = \frac{n!}{(n+3)!} \quad \text{"NON MI ACCORGO DI UNA COSA"}$$

$$\text{RAPPORTO} \quad b_{(n+1)} = \frac{(n+1)!}{((n+1)+3)!} = \frac{(n+1)!}{(n+4)!}$$

$$(n+4)! = \frac{1 \cdot 2 \cdot 3 \cdot \dots \cdot n (n+1)(n+2)(n+3)(n+4)}{(n+3)!} = (n+3)! (n+4)$$

$$\lim_{n \rightarrow +\infty} \frac{b_{n+1}}{b_n} = \lim_{n \rightarrow +\infty} \frac{(n+1)!}{(n+4)!} \cdot \frac{(n+3)!}{n!} = \lim_{n \rightarrow +\infty} \frac{n! (n+1)}{(n+3)! (n+4)} \cdot \frac{(n+3)!}{n!}$$

$$\lim_{n \rightarrow +\infty} \frac{(n+1)}{(n+4)} = 1 \quad \text{(PURTROPPO)} \rightarrow \text{NON POSSO DIRE NULLA}$$

$$\text{RIPRENDO} \quad b_n = \frac{n!}{(n+3)!} \quad \text{E CI LAVORO}$$

$$b_n = \frac{1}{n(n+1)(n+2)} \sim \frac{1}{n^3}$$

$$b_m = \frac{m!}{m!(m+1)(m+2)(m+3)} \sim C_m = \frac{1}{m^3}$$

$m^3$

$\alpha = 3 > 1$  CONV.

$$3) \sum_{n=2}^{+\infty} \frac{\sin n + (-1)^n \cdot n}{n^2 \cdot e_m n}$$

PROF  $\ln$  LO SCRIVE  
 $\log$  BASE "e"

OSSERVO  $-1 \leq \sin n \leq 1$

OSSERVO  $(-1)^n \cdot n \begin{cases} \rightarrow +\infty & n \text{ PARI} \\ \rightarrow -\infty & n \text{ DISPARI} \end{cases} \quad n \rightarrow \infty$

IN OGNI CASO  $\sin n + (-1)^n \cdot n \sim (-1)^n \cdot n$

$$\sum_{n=2}^{+\infty} \frac{(-1)^n \cdot n}{n^2 e_m n} \Rightarrow \sum_{n=2}^{+\infty} (-1)^n \cdot \frac{1}{n \cdot e_m n}$$

SERIE A SEGNI ALTERNI  $(-1)^n$

CRITERIO LEIBNIZ

$$\sum (-1)^n a_n$$

$$1) \lim_{n \rightarrow \infty} a_n = 0$$

2) LA SUCCESSIONE  $a_n$  DEVE ESSERE A TERMINI  
POSITIVI E DECRESCENTE

SE SONO VERIFICATI 1) e 2) LA SERIE CONVERGE  
(ORALE)

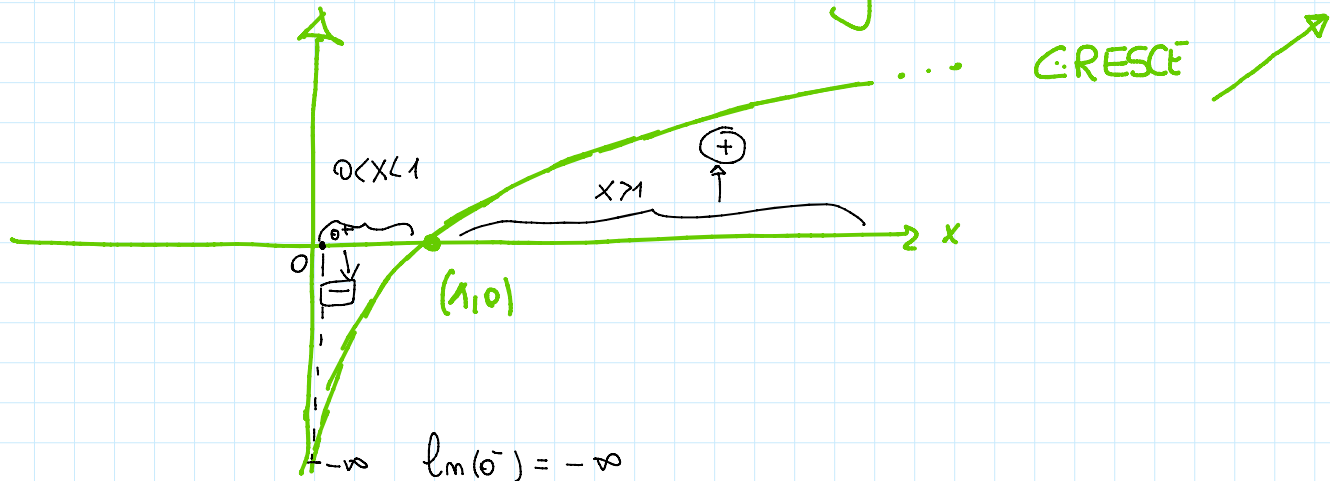
$$\sum_{n=2}^{+\infty} (-1)^n \frac{1}{n \cdot e_m n}$$

$$1) \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = 0 \quad \text{PUNTO 1 OK}$$

$$1) \lim_{n \rightarrow \infty} \frac{1}{n \ln n} = \frac{1}{\infty} = 0 \quad \text{PUNTO 1 OK}$$

$$2) \frac{1}{n \cdot \ln n} \text{ TERMINI POSITIVI}$$

GRAFICO LOGARITMO NATURALE  $y = \ln x$



$$\sum_{n=2}^{\infty} (-1)^n \frac{1}{n \cdot \ln n} \rightarrow \ln n \text{ POSITIVO}$$

$n \geq 1$

$a_n$  DECRESCERE? in  $a_n$  C'E'  $n$  CRESCERE  
 $\ln n$  CRESCERE  
 $n \cdot \ln n$  CRESCERE

Se che  $n \cdot \ln(n)$  CRESCERE  $\Rightarrow \frac{1}{n \ln n}$  DECRESCERE (INVERSO)

HO DIMOSTRATO CHE  $a_n = \frac{1}{n \cdot \ln(n)}$  DECRESCERE

ED E' A TERMINI POSITIVI.

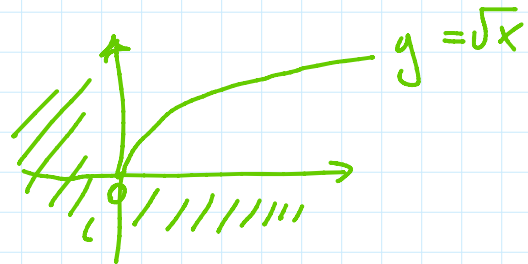
PER IL CRITERIO DI LEIBNIZ LA SERIE

PER IL CRITERIO DI LEIBNIZ LA SERIE  
CONVERGE

VI LASCIO

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$$

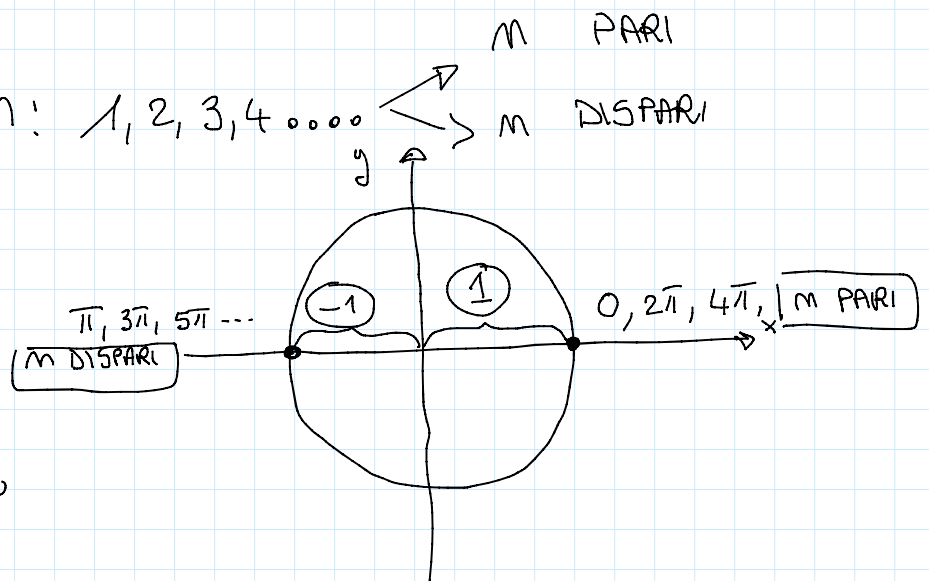
GRAFICO  $y = \sqrt{x}$



D:  $x \geq 0$   
SEMPRE  $\geq 0$   
CRESCe

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n + \sqrt{n}}$$

$\cos(n\pi)$   $n: 1, 2, 3, 4, \dots$



$$\cos(n\pi) = (-1)^n$$

$$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n + \sqrt{n}}$$

1)  $a_n = \frac{1}{n + \sqrt{n}}$   $a_n \rightarrow 0$  per  $n \rightarrow \infty$

2)  $\boxed{n + \sqrt{n} \text{ POSITIVA E CRESCe}}$

2)  $(n + \sqrt[n]{n})$  POSITIVA E CRESCE

$$\Rightarrow \frac{1}{n + \sqrt[n]{n}} \text{ DECRESCERE}$$

PER LEIBNIZ LA SERIE CONVERGE

$$\sum_{n=1}^{\infty} \left( \sqrt[n]{n} - 1 \right)^n \quad \text{NO LEIBNIZ, NON HO } (-1)^n$$

$\sqrt[n]{n}$  L'ESPOLENTE DI "TUTTO" USO CRITERIO RADICE

$$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} \begin{cases} < 1 & \text{CONVERGE} \\ > 1 & \text{DIVERGE} \\ = 1 & ? \end{cases}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\left( \sqrt[n]{n} - 1 \right)^n}$$

$$\lim_{n \rightarrow \infty} \left( \sqrt[n]{n} - 1 \right)$$

$$\lim_{n \rightarrow \infty} \left( e^{\frac{1}{n}} - 1 \right) = \lim_{n \rightarrow \infty} \left( e^{\frac{\ln n}{n}} - 1 \right)$$

$$n^{\frac{1}{n}} = e^{\frac{\ln n}{n}} \quad \text{INVERSI}$$

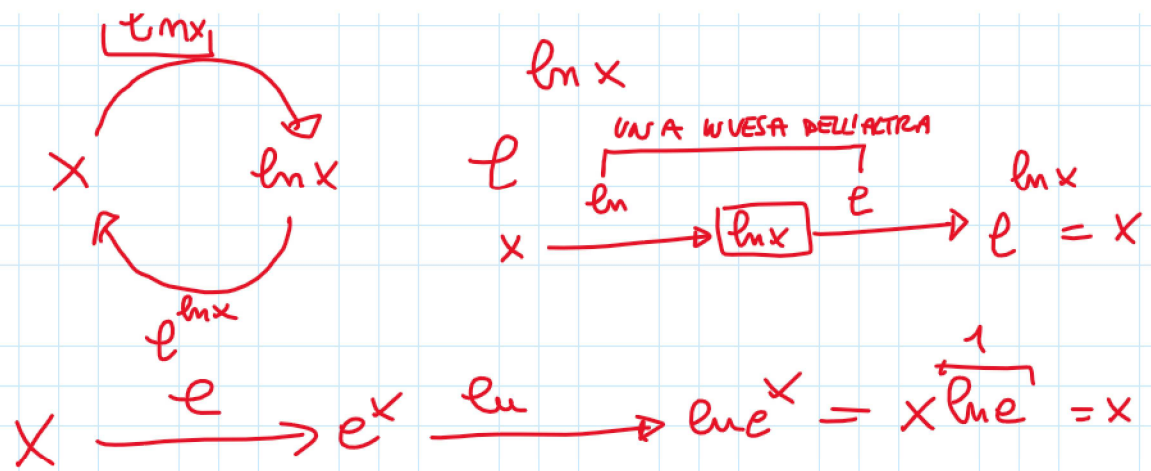
$$\lim_{n \rightarrow \infty} \left( e^{\frac{\ln n}{n}} - 1 \right) = \lim_{n \rightarrow \infty} \left( e^{\frac{1}{n} \ln n} - 1 \right)$$

$$\lim_{n \rightarrow \infty} \left( e^{\frac{\ln n}{n}} - 1 \right) = \lim_{n \rightarrow \infty} \left( e^{\frac{1}{n} \ln n} - 1 \right) = 0$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = 0 < 1 \Rightarrow \text{CRITERIO RADICE LA SERIE CONVERGE.}$$

LOGARITMO ED ESPONENZIALE SONO  
UNO L'INVERSO DELL'ALTRO

$\ln x$   $e^x$



espon.  $\leftarrow e^{\ln[f(x)]} \iff f(x)$

$\ln e^{[f(x)]} \iff f(x)$

ALCUNI LIMITI

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x^2 \cdot \sin \frac{1}{x}} \quad \text{FATTO}$$

$$\sin t = t - \frac{t^3}{3!} \dots$$

$$\lim_{x \rightarrow 0^+} \frac{[\ln(1+e^{\frac{1}{x}})] \cdot \sin x^7}{\left(\frac{1}{1-x^2}\right)^{\sin^2 x} - 1 - x^4}$$

$$x \rightarrow 0^+ \quad \sin x^7 \sim x^7$$

$$x \rightarrow 0^+ \quad \sin x \sim x \quad \sin^2 x \sim x^2$$

$$x \rightarrow 0^+ \quad e^{\frac{1}{x}} = e^{\frac{1}{0^+}} = e^{+\infty}$$

$$\ln(1+e^{\frac{1}{x}}) \xrightarrow{L'H} \ln(+\infty) = +\infty$$

$$\ln(1+f(x)) \sim f(x) \text{ se } f(x) \rightarrow 0$$

$$\ln \left( e^{\frac{1}{x}} \cdot \left( \frac{1}{e^{\frac{1}{x}}} + 1 \right) \right)$$

$$\ln e^{\frac{1}{x}} + \ln \left( 1 + \frac{1}{e^{\frac{1}{x}}} \right) = \frac{1}{x} + \frac{1}{e^{\frac{1}{x}}}$$

$\rightarrow f(x) \text{ per } x \rightarrow 0^+ \Rightarrow \frac{1}{e^{\frac{1}{0^+}}} = \frac{1}{e^{+\infty}} = \frac{1}{\infty} = 0$

$\rightarrow f(x) \rightarrow 0 \text{ per } x \rightarrow 0^+$

$$\lim_{x \rightarrow 0^+} \left[ \frac{1}{x} + \frac{1}{e^{\frac{1}{x}}} \right] \cdot x^7$$

$$\lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x} \right)^x = e$$

$$\lim_{x \rightarrow 0} \left( 1 + x \right)^{\frac{1}{x}} = e$$

$$\boxed{\left( \frac{1}{1-x^2} \right)^{x^2} = e^{\ln \left( \frac{1}{1-x^2} \right)^{x^2}} = e^{x^2 \cdot \ln \left( \frac{1}{1-x^2} \right)}} = e$$

$$= e^{x^2 \left( \underbrace{\ln 1}_{\rightarrow 0} - \ln(1-x^2) \right)}$$

$$= e^{x^2 (0 - (-x^2))} = e^{x^4}$$

$\ln(1-x^2) \sim -x^2$   
 $\hookrightarrow f(x) \rightarrow 0 \quad x \rightarrow 0$

$$\lim_{x \rightarrow 0^+} \frac{\left[ \frac{1}{x} + \frac{1}{e^{\frac{1}{x}}} \right] \cdot x^7}{e^{\frac{1}{x^4}} - 1 - x^4} = \frac{\boxed{\frac{1}{x} \cdot x^7}}{\boxed{\phantom{0}}} = \frac{1}{e^{\frac{1}{0^+}}} = \frac{1}{e^{+\infty}} = \frac{1}{\infty} = 0$$

$\hookrightarrow$  TAYLOR  
 $e^t = \dots \quad t = x^4$

$$e^t = 1 + t + \frac{t^2}{2}$$

$$\cancel{1 + x^4} + \frac{x^8}{2} - \cancel{1 - x^4}$$

$$\frac{\cancel{x^6}}{\frac{x^8}{2}} = \infty$$

$$e^t = 1 + t + \frac{t^2}{2} + \frac{t^3}{3!} = \cancel{1 + x^4} + \frac{x^8}{2} + \frac{x^{12}}{6} - \cancel{1 - x^4}$$

$$\frac{x^6}{x^6(x^2 + \underline{x^4})}$$

$$\lim_{x \rightarrow 0^+} \ln(1 + e^{\frac{1}{x}}) \cdot \sin x^7$$

$$\lim_{x \rightarrow 0^+} \frac{\ln\left(\frac{1}{1-x^2}\right) \cdot \sin x}{\left(\frac{1}{1-x^2}\right)^{\sin^2 x} - 1 - x^4}$$

PARTITE DAL DENOMINATORE  $\sin x \sim x$   $\sin^2 x \sim x^2$

$$\left(\frac{1}{1-x^2}\right)^{x^2} = e^{\ln\left(\frac{1}{1-x^2}\right)^{x^2}} = e^{x^2 \ln\left(\frac{1}{1-x^2}\right)}$$

$$e^{x^2(\ln 1 - \ln(1-x^2))} \quad \ln(1-x^2) = -x^2$$

$$e^{x^2(0 - (-x^2))} = e^{x^4}$$

$$\Delta = e^{x^4} - 1 - x^4$$

$$e^t = 1 + t + \frac{t^2}{2} + \frac{t^3}{6} + \frac{t^4}{24} + \frac{t^5}{120} + \dots$$

$$e^{x^4} = 1 + x^4 + \frac{x^8}{2!} + \dots$$

$$\Delta: 1 + x^4 + \frac{x^8}{2} - 1 - x^4$$

$$\Delta = \frac{x^8}{2} + o(x^8)$$

$$\sin x^7 =$$

$$\boxed{\sin t = t - \frac{t^3}{3!} + \dots}$$

## INTEGRALI

$$1) \int f'(x) \cdot (f(x))^m dx = \frac{(f(x))^{m+1}}{(m+1)} + C$$

$$\text{ES} \quad \int \underbrace{3x^2}_{f'(x)=3x^2} \cdot \underbrace{(x^3+2)^5}_{f(x)=x^3+2} dx = \frac{(x^3+2)^6}{6} + C$$

$m=5$

$$\text{ES} \quad \int \underbrace{e^x}_{f'(x)=e^x} \cdot \underbrace{(e^x+7)^2}_{f(x)=e^x+7} dx = \frac{(e^x+7)^3}{3} + C$$

$$\underbrace{f(x) = e^x + 7 \quad f'(x) = e^x}_{n=2} \quad \frac{\quad}{3} + C$$

$$2) \int f'(x) \cdot e^{f(x)} dx = e^{f(x)} + C$$

$$ES \int \frac{f'(x)}{4} \cdot e^{\frac{f(x)}{4x+6}} dx = e^{\frac{f(x)}{4x+6}} + C$$

$$3) \int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$$

$$ES \int \frac{\frac{f'(x)}{-\sin x}}{\frac{\cos x}{f(x)}} dx = \ln |f(x)| + C$$

$$\int -\tan x dx = \ln |f(x)| + C$$

$$4) \int f'(x) \cdot \sin f(x) dx = -\cos f(x) + C$$

$$ES \int \underbrace{e^x}_{f'(x)} \cdot \sin(\underbrace{e^x+2}_{f(x)}) dx = -\cos(e^x+2) + C$$

$$5) \int f'(x) \cdot \cos f(x) dx = \sin f(x) + C$$

$$6) \int \frac{f'(x)}{1 + |f(x)|^2} dx = \arctg f(x) + C$$

$$b) \int \frac{1}{1 + (f(x))^2} dx = \arctan f(x) + C$$