

METTERE LA CONICA IN FORMA CANONICA E CLASSIFICARLA

$$11x^2 + 16xy - y^2 + 50x + 50y + 35 = 0$$

$$A = \begin{pmatrix} "x^2" & "xy" & "x" \\ "xy" & "y^2" & "y" \\ "x" & "y" & "t.m" \end{pmatrix}$$

"NUMERI"

SIMMETRICA

se $\det A = 0$
 $\Rightarrow$ DEGENERE

se $\det A \neq 0$
 $\Rightarrow$ NON DEGENERE

$$A = \begin{pmatrix} 11 & 8 & 25 \\ 8 & -1 & 25 \\ 25 & 25 & 35 \end{pmatrix}$$

$$A_{33} = \begin{pmatrix} 11 & 8 \\ 8 & -1 \end{pmatrix}$$

ELIMINO
RIGA 3
COLONNA 3

$\det A_{33} > 0$ ELLISSE
 $\det A_{33} < 0$ IPERBOLE
 $\det A_{33} = 0$ PARABOLA

CLASSIFICATA

$$\det A_{33} = -11 - 64 < 0 \quad \text{IPERBOLE}$$

1) CERCO GLI AUTOVALORI DI A_{33}

$$\det \begin{pmatrix} 11 - \lambda & 8 \\ 8 & -1 - \lambda \end{pmatrix} = 0 \rightarrow (11 - \lambda)(-1 - \lambda) - 64 = 0$$

$$-11 - 11\lambda + \lambda + \lambda^2 - 64 = 0 \quad \lambda^2 - 10\lambda - 75 = 0$$

$$(\lambda - 15)(\lambda + 5) = 0$$

$$\lambda_1 = 15$$

$$\lambda_2 = -5$$

$\lambda_1 = 15$ CERCO AUTOVETTORE METTO 15 AL POSTO DI λ IN $A_{33} - \lambda I$

$$\begin{pmatrix} -4 & 8 \\ 8 & -16 \end{pmatrix} \leftarrow \text{PROPORZIONALI (DEVE ACCADERE SEMPRE)}$$

$$\begin{pmatrix} -4 & 8 \\ 0 & 0 \end{pmatrix} (:4) \quad \begin{pmatrix} -1 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -x + 2y = 0 \\ 0 = 0 \end{cases} \quad \begin{cases} x = 2y \\ y = t \end{cases} \quad \begin{cases} x = 2t \\ y = t \end{cases}$$

$$(2t, t) \quad t(2, 1) \quad \vec{v}_1 = (2, 1)$$

MI OCCORRE IL VETTORE $\vec{u}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} \quad \|\vec{v}_1\| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$$\vec{u}_1 = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right)$$

MOMENTO DELICATO

RICAVO $\vec{u}_2$ DA $\vec{u}_1$ SCAMBIAVO LE COMPONENTI DI $\vec{u}_1$ E IL SEGNO AD UNA SOLA DI ESSE

$$\vec{u}_2 = \begin{pmatrix} -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \end{pmatrix} \quad \text{QUALE SCELGO?}$$

SCRIVO P, LA MATRICE DEGLI AUTORESORI IN COLONNA

$$P_1 = \begin{pmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}$$

$$P_2 = \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \end{pmatrix}$$

$$P_1 \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}$$

$$P_2 \begin{pmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \end{pmatrix}$$

$$\det P_1 = \frac{2}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} = \frac{4}{5} + \frac{1}{5} = 1 \quad \text{OK}$$

ROTAZIONE

$$\det P_2 = -\frac{4}{5} - \frac{1}{5} = -1$$

180
RIBALTAMENTO

SCELGO

$$P \begin{pmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}$$

1^a FORMULA

$$\begin{pmatrix} x \\ y \end{pmatrix} = P \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\begin{cases} x = \frac{2}{\sqrt{5}} \bar{x} - \frac{1}{\sqrt{5}} \bar{y} \\ y = \frac{1}{\sqrt{5}} \bar{x} + \frac{2}{\sqrt{5}} \bar{y} \end{cases}$$

COSA ACCADE

$$11x^2 + 16xy - y^2 + 50x + 50y + 35 = 0$$

$$\lambda_1 \bar{x}^2 + \lambda_2 \bar{y}^2$$

SOSTITUISCO DALLE FORMULE

$$15\bar{x}^2 - 5\bar{y}^2 + 50\left(\frac{2}{\sqrt{5}}\bar{x} - \frac{1}{\sqrt{5}}\bar{y}\right) + 50\left(\frac{1}{\sqrt{5}}\bar{x} + \frac{2}{\sqrt{5}}\bar{y}\right) + 35 = 0$$

$$15\bar{x}^2 - 5\bar{y}^2 + \frac{100}{\sqrt{5}}\bar{x} - \frac{50}{\sqrt{5}}\bar{y} + \frac{50}{\sqrt{5}}\bar{x} + \frac{100}{\sqrt{5}}\bar{y} + 35 = 0 \quad (05)$$

$$15x^2 - 5y^2 + \frac{100}{\sqrt{5}}x - \frac{50}{\sqrt{5}}y + \frac{50}{\sqrt{5}}x + \frac{100}{\sqrt{5}}y + 35 = 0 \quad (105)$$

$$3x^2 - y^2 + \frac{20}{\sqrt{5}}x - \frac{10}{\sqrt{5}}y + \frac{10}{\sqrt{5}}x + \frac{20}{\sqrt{5}}y + 7 = 0$$

$$3x^2 - y^2 + \frac{30}{\sqrt{5}}x + \frac{10}{\sqrt{5}}y + 7 = 0$$

RAZIONALIZZAZIONE

$$\frac{30}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{30\sqrt{5}}{5} = 6\sqrt{5}$$

$$\frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$

$$3x^2 - y^2 + 6\sqrt{5}x + 2\sqrt{5}y + 7 = 0$$

PER LA TRASLAZIONE USO IL COMPLETAMENTO DEL QUADRATO

$$3x^2 + 6\sqrt{5}x$$

DEVO RACCOGLIERE SEPPURE IL COEFFICIENTE DI GRADO MAX

$$3(1x^2 + 2\sqrt{5}x) = 3(x^2 + 2\sqrt{5}x + 5 - 5)$$

$$\downarrow :2$$

$$\sqrt{5} \rightarrow (\)^2 = 5$$

$$3((x + \sqrt{5})^2 - 5)$$

$$3(x + \sqrt{5})^2 - 15$$

$$-y^2 + 2\sqrt{5}y$$

DEVO RACCOGLIERE -

$$-(1y^2 - 2\sqrt{5}y) = - (y^2 - 2\sqrt{5}y + 5 - 5)$$

$$\downarrow :2$$

$$\sqrt{5} \rightarrow (\)^2 = 5$$

$$-((y - \sqrt{5})^2 - 5)$$

$$-(y - \sqrt{5})^2 + 5$$

$$3x^2 - y^2 + 6\sqrt{5}x + 2\sqrt{5}y + 7 = 0$$

$$3(x+\sqrt{5})^2 - 15 - (y-\sqrt{5})^2 + 5 + 7 = 0$$

TRASLAZIONE

$$\begin{cases} x' = x + \sqrt{5} \\ y' = y - \sqrt{5} \end{cases}$$

$$3x'^2 - 15 - y'^2 + 5 + 7 = 0 \quad x' = x \quad y' = y$$

$$3x^2 - y^2 = 3 \quad (:3)$$

$$x^2 - \frac{y^2}{3} = 1 \quad \text{IPERBOLE}$$

$$x^2 + 2xy + y^2 + 9\sqrt{2}x - 5\sqrt{2}y + 2 = 0$$

$$A_{33} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \det A_{33} = 0 \Rightarrow \text{PARABOLA} \quad (\lambda = 0)$$

$$\det \begin{pmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{pmatrix} = (1-\lambda)^2 - 1 = 0$$

$$1 - 2\lambda + \lambda^2 - 1 = 0$$

$$\lambda(\lambda - 2) = 0 \Rightarrow \begin{cases} \lambda = 0 \\ \lambda = 2 \end{cases}$$

$$\lambda_1 = 0 \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} p \\ 0 \end{pmatrix}$$

$$\begin{cases} X+Y=0 \\ 0=0 \end{cases} \quad \begin{cases} X=-Y \\ Y=t \end{cases} \quad \begin{cases} X=-t \\ Y=t \end{cases} \quad (-t, t)$$

$$t(-1, 1) \quad \vec{v}_1 = (-1, 1) \quad \|\vec{v}_1\| = \sqrt{2}$$

$$u_1 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$u_2 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\det \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = -\frac{1}{2} - \frac{1}{2} = -1$$

No

SCAMBIO I SEGUI ALLA 1^a COLONNA

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\begin{cases} X = \frac{1}{\sqrt{2}}X + \frac{1}{\sqrt{2}}Y \\ Y = -\frac{1}{\sqrt{2}}X + \frac{1}{\sqrt{2}}Y \end{cases}$$

$$X^2 + 2XY + Y^2 + 9\sqrt{2}X - 5\sqrt{2}Y + 2 = 0$$

$$\underbrace{0X^2 + 2Y^2}_{\substack{\uparrow \\ \lambda_1}} + \underbrace{9\sqrt{2}}_{\substack{\uparrow \\ \lambda_2}} \left(\frac{1}{\sqrt{2}}X + \frac{1}{\sqrt{2}}Y \right) - 5\sqrt{2} \left(-\frac{1}{\sqrt{2}}X + \frac{1}{\sqrt{2}}Y \right) + 2 = 0$$

$$2Y^2 + 9X + 9Y + 5X - 5Y + 2 = 0$$

$$2Y^2 + 4Y + 14X + 2 = 0 \quad (:2)$$

$$Y^2 + 2Y + 7X + 1 = 0$$

$$Y^2 + 2Y + 1 + 7X = 0$$

$$(Y+1)^2 + 7X = 0$$

$$(y+1)^2 + 7x = 0$$

$$\begin{cases} y' = y+1 \\ x' = x \end{cases}$$

$$y'^2 + 7x'$$

$$x' = x \quad y' = y$$

$$y^2 + 7x = 0$$

$$x = -\frac{1}{7}y^2$$