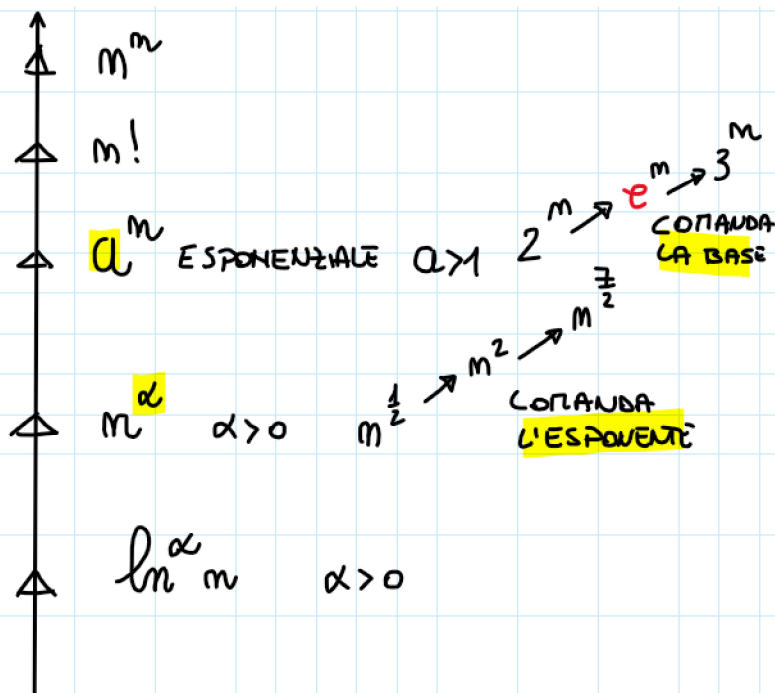


$$n \rightarrow \infty$$

ORDINI DI ∞



$$\lim_{n \rightarrow +\infty} \frac{3n - 7n^2 + 1}{5n - 2n^2 + 6} \quad \text{F.I.}$$

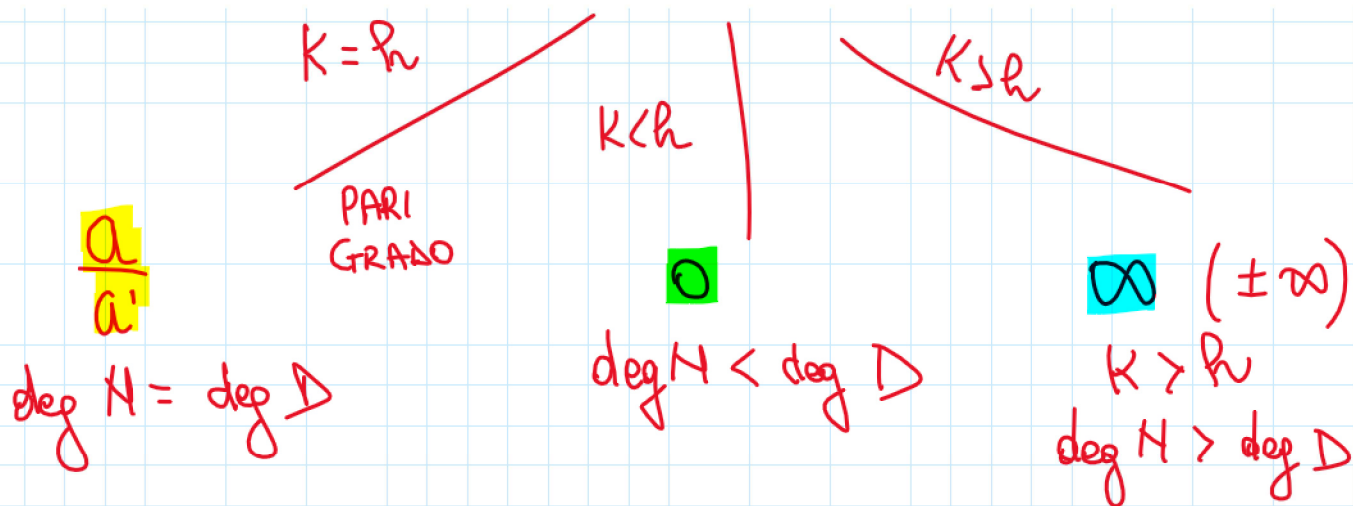
RACCOLGO IN N e D LA n DI GRADO MASSIMO

$$\lim_{n \rightarrow +\infty} \frac{n^2 \cdot \left(\frac{3}{n} - 7 + \frac{1}{n^2} \right)}{n^2 \cdot \left(\frac{5}{n} - 2 + \frac{6}{n^2} \right)} = \frac{-7}{-2} = \frac{7}{2}$$

$$\lim_{n \rightarrow +\infty} \frac{3n - 7n^2 + 1}{5n - 2n^2 + 6} \sim \lim_{n \rightarrow +\infty} \frac{-7n^2}{-2n} = \frac{7}{2}$$

REGOLA

$$\lim_{n \rightarrow +\infty} \frac{a n^k + b n^{k-1} + \dots}{a' n^h + b' n^{h-1} + \dots}$$



ϕ

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 - 1} - (n + 1) \right] = \infty - \infty$$

USO IL PRODOTTO NOTEVOLE $(A+B)(A-B) = A^2 - B^2$

SE NEL LIMITE C'È $A - B$ ALLORA MOLTIPLICO E DIVIDO PER $A + B$ E VICEVERSA

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 - 1} - (n + 1) \right] \cdot \frac{\sqrt{n^2 - 1} + (n + 1)}{\sqrt{n^2 - 1} + (n + 1)}$$

$A + B$

$$\lim_{n \rightarrow \infty} \frac{(\sqrt{n^2 - 1})^2 - (n + 1)^2}{\sqrt{n^2 - 1} + (n + 1)}$$

$A^2 - B^2$

$$\lim_{n \rightarrow \infty} \frac{n^2 - 1 - n^2 - 2n - 1}{\sqrt{n^2 - 1} + n + 1}$$

n

$$-2n - 2$$

Quindi la m...

$$\lim_{n \rightarrow \infty} \frac{-2n - 2}{\sqrt{n^2 - 1} + n + 1}$$

RACCOLGO LA n
DI GRADO MAX $\left(\frac{\infty}{\infty}\right)$

$$\lim_{n \rightarrow \infty} \frac{n \left(-2 - \frac{2}{n} \right)}{\sqrt{n^2 \left(1 - \frac{1}{n^2} \right)} + n + 1}$$

$$\lim_{n \rightarrow \infty} \frac{n \left(-2 - \frac{2}{n} \right)}{\underbrace{n \sqrt{1 - \frac{1}{n^2}}}_{\rightarrow 1} + \underbrace{n}_{\rightarrow \infty} + \underbrace{1}_{\rightarrow 0}}$$

RACCOLGO n

$$\lim_{n \rightarrow \infty} \frac{\cancel{n} \left(-2 - \frac{2}{\cancel{n}} \right)}{\cancel{n} \left(\sqrt{1 - \frac{1}{\cancel{n}^2}} + 1 + \frac{1}{\cancel{n}} \right)} = \frac{-2}{\sqrt{1} + 1} = \frac{-2}{2} = -1$$

PER VOI

$$\lim_{n \rightarrow +\infty} \left(\sqrt{n^2 + 3n + 1} - n \right) \quad \left[\frac{3}{2} \right]$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 2} - n \right) \quad [0]$$

$$\lim_{n \rightarrow \infty} \left(\frac{n}{\sqrt{n+2}} - \frac{n}{\sqrt{n+1}} \right)$$

SUGGERIMENTO:
FAI IL m.c.m.
" OPPURE RAZIONALIZZAT
E POI IL m.c.m. "

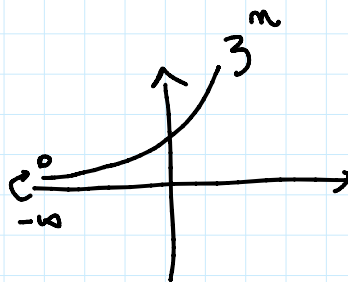
$$\lim_{n \rightarrow +\infty} \frac{2^{\frac{1}{n}} + n^2 + 3^{-n}}{0.6n + 1 + n}$$

$$n \rightarrow +\infty \quad \lim_{n \rightarrow +\infty} n^6 + 2 + n$$

$$n \rightarrow +\infty \quad 2^{\frac{1}{n}} \rightarrow 2^0 = 1$$

$$n^2 \rightarrow \infty$$

$$3^{-n} \rightarrow 3^{-\infty}$$



$$3^{-\infty} = 0$$

$$3^{-n} = \frac{1}{3^n} = \frac{1}{\infty} = 0$$

$$\sim \lim_{n \rightarrow \infty} \frac{n^2}{n} = \infty$$

LIMITE NOTEVOLE "e"

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n = \left(1 + \frac{1}{\infty} \right)^{\infty} = 1^{\infty}$$

F.I.
↓
0

SI DIMOSTRA

$$\lim_{n \rightarrow \pm \infty} \left(1 + \frac{1}{n} \right)^n = e$$

AMPLIANDO

$$\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{a_n} \right)^{a_n} = e$$

A PATTO CHE $a_n \rightarrow \pm\infty$ QUANDO $n \rightarrow \pm\infty$

$$\lim_{n \rightarrow \infty} \left(\frac{n+2}{n-1} \right)^{2n} = 1^\infty \text{ F.I.}$$

DEVO RICREARE LA SITUAZIONE

$$\left(1 + \frac{1}{a_n} \right)^{a_n}$$

$$\frac{n+2}{n-1}$$

AL NUMERATORE SOMMO E SOTTRAGGO DELLE QUANTITÀ SCELTE DA ME PER RIPRODURRE IL DENOMINATORE

$$\frac{\overbrace{n-1} + \overbrace{+1} + \overbrace{+2}}{n-1} = \frac{n-1+3}{n-1} = \frac{n-1}{n-1} + \frac{3}{n-1}$$

$$\underset{\text{OK}}{1} + \underset{\text{OK}}{\frac{3}{n-1}} = \underset{\text{OK}}{1} + \frac{\overset{\text{OK}}{1}}{\underset{3}{\frac{n-1}{3}}}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{\boxed{\frac{n-1}{3}}} \right)^{2n \quad a_n?}$$

$a_n \rightarrow \infty \quad n \rightarrow \infty$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{\frac{n-1}{3}} \right)^{\frac{n-1}{3}} \cdot \boxed{\frac{3}{n-1} \cdot 2n}$$

$$\lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{\frac{n-1}{3}} \right)^{\frac{n-1}{3}} \right] \left(\frac{6n}{n-1} \right) \quad n \rightarrow \infty$$

$$e^6$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^2 + 2}{n^2 + n + 1} \right)^{2n} = 1^\infty \text{ F.I.}$$

$$\frac{n^2 + 2}{n^2 + n + 1} = \frac{n^2 + n + 1 - n - 1 + 2}{n^2 + n + 1}$$

$$\frac{n^2 + n + 1}{n^2 + n + 1} + \frac{-n + 1}{n^2 + n + 1} = 1 + \frac{1}{\frac{n^2 + n + 1}{-n + 1}}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{\frac{n^2 + n + 1}{-n + 1}} \right)^{\frac{n^2 + n + 1}{-n + 1} \cdot \frac{-n + 1}{n^2 + n + 1} \cdot 2n}$$

$$\lim \left[\left(1 + \frac{1}{\frac{n^2 + n + 1}{-n + 1}} \right)^{\frac{n^2 + n + 1}{-n + 1}} \right]^{\frac{-2n^2 + 2n}{n^2 + n + 1}} = e^{-2}$$

$$\lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{\frac{n^2+n+1}{-n+1}} \right) \right] = e^{-}$$

$\hookrightarrow e$

CRITERIO DEL RAPPORTO PER IL CALCOLO DEL LIMITE DI UNA SUCCESSIONE A TERMINI POSITIVI

- 0) STO CALCOLANDO $\lim_{n \rightarrow \infty} a_n$????????
- 1) HO UNA SUCCESSIONE a_n
- 2) CALCOLO $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = l$ $\begin{cases} \text{se } l < 1 & \lim_{n \rightarrow \infty} a_n = 0 \\ \text{se } l > 1 & \lim_{n \rightarrow \infty} a_n = +\infty \end{cases}$

FATTORIALE

$$7! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$$

$$\downarrow$$

$$7! = 6! \cdot 7$$

$$7! = 5! \cdot 6 \cdot 7$$

$$\begin{aligned} (m+1)! &= \underbrace{1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot m}_{m!} \cdot (m+1) \\ &= m! (m+1) \\ &= (m-1)! \cdot m \cdot (m+1) \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{n!}{n^n} = \frac{\infty}{\infty} \text{ F.I. } 0 \text{ ??????}$$

CRITERIO RAPPORTO $a_n = n!$ calcola

CRITERIO RAPPORTO

$$a_n = \frac{n!}{n^n} \quad \text{CALCOLO}$$

$$a_{(n+1)} = \frac{(n+1)!}{(n+1)^{(n+1)}}$$

$$\lim_{n \rightarrow +\infty} \frac{a_{(n+1)}}{a_n} = \lim_{n \rightarrow +\infty} \frac{\frac{(n+1)!}{(n+1)^{(n+1)}}}{\frac{n!}{n^n}}$$

$$= \lim_{n \rightarrow +\infty} \frac{(n+1)!}{(n+1)^{(n+1)}} \cdot \frac{n^n}{n!}$$

"LO SEMPLIFICHI"

$$\lim_{n \rightarrow +\infty} \frac{\cancel{n!} \cdot \cancel{(n+1)}}{(n+1)^n \cdot \cancel{(n+1)^1}} \cdot \frac{n^n}{\cancel{n!}}$$

$$\lim_{n \rightarrow +\infty} \frac{n^n}{(n+1)^n}$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$\lim_{n \rightarrow +\infty} \left(\frac{n}{n+1}\right)^n \xrightarrow{1^\infty} \lim_{n \rightarrow +\infty} \left(\frac{1}{\frac{n+1}{n}}\right)^n$$

$$\lim_{n \rightarrow +\infty} \left(\frac{1}{\frac{n}{n} + \frac{1}{n}}\right)^n$$

$$\frac{n}{n+1} = \frac{n+1-1}{n+1}$$

$$\frac{n+1}{n+1} + \frac{-1}{n+1}$$

$$\lim_{n \rightarrow +\infty} \frac{1}{1 + \frac{1}{n}} = \frac{1}{e}$$

$$\frac{n+1}{n+1} + \frac{-1}{\underline{n+1}}$$

$$1 + \frac{1}{\underline{-n-1}}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\left(1 + \frac{1}{n}\right)^n} = \frac{1}{e}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{\underline{-n-1}}\right)^n$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{\underline{-n-1}}\right)^{(-n-1) \cdot \left(\frac{1}{-n-1}\right) \cdot n}$$

$$\lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{\underline{-n-1}}\right)^{-n-1} \right]^{\boxed{\frac{n}{-n-1}}} = e^{\boxed{-1}} = \frac{1}{e}$$

SI CONE $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{1}{e} < 1$ ALLORA

$$\lim_{n \rightarrow \infty} \frac{n!}{n^n} = \underline{\underline{0}}$$

PER VOI $\lim_{n \rightarrow \infty} \frac{2^n \cdot n!}{n^n} \quad [=0]$

$$\lim_{n \rightarrow \infty} \left(\frac{n^2 + 2n}{n^2 - 3} \right)^{-5n} \quad [e^{-10}]$$