LIMITI DI FUNZIONE

venerdì 13 ottobre 2023 08:42

DAI LIMITI NOTEVOLI DERIVANO LE "STIME ASINTOTICHE"

$x \to 0$	$f(x) \to 0$	$a_n \to 0$ SE $n \to \infty$
$\sin x \sim x$	$\sin(f(x)) \sim f(x)$	$\sin a_n \sim a_n$
$1-\cos x \sim \frac{1}{2}x^2$	$1-\cos(f(x)) \sim \frac{1}{2}(f(x))^2$	$1-\cos a_n \sim \frac{1}{2}(a_n)^2$
$e^x - 1 \sim x$	$e^{f(x)} - 1 \sim f(x)$	$e^{a_n} - 1 \sim a_n$
$\ln(1+x) \sim x$	$\ln(1+f(x)) \sim f(x)$	$\ln(1+a_n) \sim a_n$
$(1+x)^\alpha - 1 \sim \alpha x$	$(1+f(x))^\alpha - 1 \sim \alpha f(x)$	$(1+a_n)^\alpha - 1 \sim \alpha a_n$
$\tan x \sim x$	$\tan f(x) \sim f(x)$	$\tan a_n \sim a_n$
$\arctan x \sim x$	$\arctan f(x) \sim f(x)$	$\arctan a_n \sim a_n$

ESERCIZIO

$$a_n = \frac{\sin\frac{1}{n}}{n^2\left(\cos\frac{1}{n} - 1\right)} \qquad n \in \mathbb{N}_0 \ \{1, 2, \dots\}$$

$n \to \infty$

$\sin\frac{1}{n} \sim \boxed{\frac{1}{n}}$ perché $\frac{1}{n} \to 0$

$\cos\frac{1}{n} - 1$ RACCOLGO $-$ $\Rightarrow -\left(-\cos\frac{1}{n} + 1\right)$

$\Rightarrow -\left(1 - \cos\frac{1}{n}\right) \sim -\frac{1}{2}\cdot\frac{1}{n^2}$

$\left(1-\cos\frac{1}{n} \sim \frac{1}{2}\left(\frac{1}{n}\right)^2\right)$

$\sim \boxed{-\frac{1}{2n^2}}$

$$a_n = \frac{\operatorname{sen}\frac{1}{n}}{n^2\left(\cos\frac{1}{n}-1\right)} \sim \frac{\frac{1}{n}}{n^2\left(-\frac{1}{2n^2}\right)} = \frac{\frac{1}{n}}{-\frac{1}{2}}$$

$$\left|\frac{2n^2}{}\right|$$

$$= \frac{1}{n}\cdot\left(-\frac{2}{1}\right) = -\frac{2}{n}$$

ASINTOTICAMENTE

$$\lim_{n\to+\infty} -\frac{2}{n} = 0$$

$n \nearrow$ $\quad -\frac{2}{n} \nearrow$ $\quad \frac{2}{n} \searrow$

$a_n \nearrow 0$ sup

$a_1 = -2$ min

ESERCIZI

1) $$\lim_{x\to 0} \frac{x\cdot \operatorname{sen} 2x}{\operatorname{sen}^2 3x} = \text{PASSAGGIO AL LIMITE} = \frac{0\cdot\operatorname{sen}0}{\operatorname{sen}^2 0} = \frac{0}{0}$$

$x\to 0$ $\quad \operatorname{sen}2x \sim 2x$

$$\operatorname{sen}^2 3x = (\operatorname{sen} 3x)^2 \sim (3x)^2 = 9x^2$$

$$\lim_{x\to 0} \frac{x\cdot 2x}{9x^2} = \lim_{x\to 0} \frac{2x^2}{9x^2} = \frac{2}{9}$$

2) $$\lim_{x\to\infty} x\cdot \operatorname{sen}\left(\frac{2}{x}\right)\cdot\cos\left(\frac{3}{x}\right) = \infty\cdot\operatorname{sen}\left(\frac{2}{\infty}\right)\cdot\cos\left(\frac{3}{\infty}\right)$$

$$= \infty\cdot\operatorname{sen}0\cdot\cos 0$$

$$= \infty\cdot 0\cdot 1$$

$$= \infty\cdot 0 \quad \text{F.I.}$$

$x\to\infty$ $\quad \operatorname{sen}\left(\frac{2}{x}\right) \sim \left(\frac{2}{x}\right)$

$\frac{2}{x} \to 0$

fun $\to 0$

$\to 0$

$\cos\frac{3}{x} = 1$

$\frac{3}{x} \to 0 \quad \cos 0$

$\lim_{x\to\infty} x \cdot \frac{2}{x} \cdot 1 = 2$

$\lim_{x\to\infty} x \cdot \operatorname{sen}\left(\frac{2}{x}\right) \cdot \cos\left(\frac{3}{x}\right)$

$x \to \infty$

↳ VORREI $\to 0$

$\frac{1}{x} = t$ SE $x \to \infty$ ALLORA $t \to 0$

$x = \frac{1}{t}$

$\lim_{t\to 0} \frac{1}{t} \cdot \operatorname{sen}\left(\frac{2}{\frac{1}{t}}\right) \cos\left(\frac{3}{\frac{1}{t}}\right)$

$2 \cdot \frac{t}{1}$ $\quad 3 \cdot \frac{t}{1}$

ok

$\lim_{t\to 0} \frac{1}{t} \cdot \operatorname{sen}(2t) \cdot \cos(3t)$

$t \to 0 \quad \operatorname{sen} 2t \sim 2t$

$\cos 3t = \cos 0 = 1$

$\lim_{t\to 0} \frac{1}{t} \cdot 2t = 2$

3) $\lim_{x\to 0} \frac{\operatorname{sen}\left(2x + \frac{\pi}{4}\right) \cdot \operatorname{sen}^2(3x)}{(\cos 2x - 1) \cdot \cos 3x} =$

$\frac{\operatorname{sen}\frac{\pi}{4} \cdot \operatorname{sen}^2 0 \to 0}{(\cos 0 - 1) \cdot \cos 0} = \frac{0}{0}$

$\cos 0 \to 1$

$x \to 0$ $\quad \operatorname{sen}\left(2x + \frac{\pi}{4}\right) = \operatorname{sen}\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\operatorname{sen}^2(3x) = \left(\operatorname{sen}(3x)\right)^2 \sim (3x)^2 = 9x^2$

$\operatorname{sen} f \sim f \quad (f \to 0)$

$\cos 2x - 1 = -\left(1 - \cos(2x)\right) \sim -\frac{1}{2}(2x)^2$

$\cos 3x = 1$

$= -\frac{1}{2} \cdot 4x^2$

$= -2x^2$

$$\lim_{x \to 0} \frac{\frac{\sqrt{2}}{2} \cdot 9x^2}{-2x^2 \cdot 1} = \frac{\frac{9\sqrt{2}}{2}}{-2} = -\frac{9\sqrt{2}}{2} \cdot \frac{1}{2} = -\frac{9\sqrt{2}}{4}$$

4) $$\lim_{x \to 0} \frac{\left(1 - \cos(\sqrt{3}x)\right) \cdot \operatorname{sen}^2 x}{x^3 \cdot \tan 2x} = \frac{0}{0}$$

$x \to 0 \quad 1 - \cos(\sqrt{3}x) \sim \frac{1}{2}\left(\sqrt{3}x\right)^2 = \frac{1}{2}\left(3x^2\right) = \frac{3}{2}x^2$

$\operatorname{sen}^2 x = (\operatorname{sen} x)^2 \sim x^2$

$\tan 2x \sim 2x$

$$\lim_{x \to 0} \frac{\frac{3}{2}x^2 \cdot x^2}{x^3 \cdot 2x} = \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}$$

5) $$\lim_{x \to 0} \frac{x^2 \cdot \operatorname{sim}\frac{1}{x}}{0 \ (x+y)} = \frac{0 \cdot \operatorname{sim}\frac{1}{0}}{0 \dots 1}$$

$$\lim_{x\to 0} \frac{x}{\ln(1+x)} = \frac{0}{\ln 1} = \frac{0\cdot \text{sen}\,\infty}{0}$$

$x \to 0 \quad \text{sen}\frac{1}{x} = K \quad -1 \le K \le 1$

$\ln(1+x) \sim x$

$$\lim_{x\to 0} \frac{x^2 \cdot K}{x} = 0 \cdot K = 0$$

6) $$\lim_{x\to 0} \frac{e^{x} - e^{-x}}{\text{sen}\,4x} \quad (\text{ESAME}) = \frac{e^0 - e^0}{\text{sen}\,0} = \frac{1-1}{0} = \frac{0}{0}$$

F. I.

1° modo

$x \to 0 \quad e^{x} - e^{-x} - 1 + 1$

• RACCOLGO −

$(e^{x} - 1) - (e^{-x} - 1) = x - (-x)$

$\sim x \qquad \sim -x \qquad x + x = 2x$

$\text{sen}\,4x \sim 4x$

$$\lim_{x\to 0} \frac{2x}{4x} = \frac{1}{2}$$

2° MODO

$x \to 0 \quad e^{x} - e^{-x}$ RACCOLGO e^{-x}

$e^{-x}\left(e^{x-(-x)} \right.$

$$e^{-x} \cdot \left(e^{x-(-x)} - 1\right)$$

$x \to 0$ $\quad e^{-x} \cdot \left(e^{2x} - 1\right)$

$e^{-x} \to e^{-0} = 1$ $\qquad e^{2x} - 1 \sim 2x$

$\sin 4x \sim 4x$ $\qquad \lim_{x \to 0} \frac{1 \cdot 2x}{4x} = \frac{1}{2}$

7) $\lim_{x \to 0} \frac{\sin 3x}{e^{2x} - e^{-2x}}$ ESAME

$x \to 0$ $\quad \sin 3x \sim 3x$

$$e^{2x} - e^{-2x} = e^{-2x}\left(e^{2x-(-2x)} - 1\right)$$

$$e^{-2x}\left(e^{4x} - 1\right)$$

$e^{-2x} \to e^{0} = 1$ $\qquad e^{4x} - 1 \sim 4x$

$\lim_{x \to 0} \frac{3x}{4x} = \frac{3}{4}$

ESAME

8) $\lim_{x \to 0} \frac{\sin x \cdot \left(x^2 \cdot e^x + 2\ln(\cos x)\right)}{x^2 \sqrt{x}}$

$x \to 0$ $\quad \sin x \sim x$

$\ln(\cos x)$

$\ln(\cos x + 1 - 1)$

$\ln(1 + f(x)) \sim f(x)$

$$\ln\left(1+(\cos x-1)\right)$$

$$f(x)=\cos x-1=-\underbrace{(1-\cos x)}_{\frac{1}{2}x^2}\sim -\frac{1}{2}x^2$$

$$\ln\left(1+\left(-\tfrac{1}{2}x^2\right)\right)\sim\left(-\tfrac{1}{2}x^2\right)$$

$$\ln(1+f(x))\sim f(x)$$

QUINDI $\ln(\cos x)\sim -\frac{1}{2}x^2$

$$\lim_{x\to 0}\frac{*\left(x^2\cdot e^x+2\left(-\frac{1}{2}x^2\right)\right)}{x^2\cdot x^{\frac{1}{2}}}$$

$$\lim_{x\to 0}\frac{\left(x^2\cdot e^x-x^2\right)}{x^{1+\frac{1}{2}}}$$

$$\lim_{x\to 0}\frac{x^2(e^x-1)}{x^{\frac{3}{2}}}\qquad e^x-1\sim x$$

$$\lim_{x\to 0}\frac{x^3}{x^{3/2}}=\lim_{x\to 0}x^{3-\frac{3}{2}}\qquad \lim_{x\to 0}x^{\frac{3}{2}}=0!$$

9) $$\lim_{x\to 0}\frac{\operatorname{sen}x-\ln\left(e^x\cdot\cos x\right)}{x\operatorname{sen}x}$$

$\log b+\log c=\log(b\cdot c)$ $\qquad \log b^n=n\log a$

$\log_a b + \log_a c = \log_a(b \cdot c)$ $\quad \log_a b^n = n \log_a a$

$\log_v v = 1$ $\quad \log_v 1 = 0$ $\quad \ln(0^+) = -\infty$

$x \to \infty \quad \ln(e^x \cdot \cos x) = \ln e^x + \ln \cos x$

$= x \ln e + \ln \cos x$

$= x - \frac{1}{2}x^2$

$$\lim_{x \to 0} \frac{x - \left(x - \frac{1}{2}x^2\right)}{x^2} = \lim_{x \to 0} \frac{x - x + \frac{1}{2}x^2}{x^2}$$

$= \frac{1}{2}$

ESERCIZI ESAME

$$\lim_{x \to +\infty} \left(\frac{x^2 + x}{x^2 - x + 1}\right)^x$$

USA $\left(1 + \frac{1}{a_n}\right)^{a_n}$ CON $f(x)$ $\left(1 + \frac{1}{f(x)}\right)^{f(x)}$

$$\lim_{x \to 0} \frac{\operatorname{sen} x + e^x + 3x^2 - 1}{x^2}$$

ACCOPPIA $e^x \in -1$

$$\lim_{x \to 0} \frac{e^{x^2} - 1}{\ln(1 + 2x + 3x^2)}$$

ACCOPPIA $2x \in 3x^2$

$$\lim_{x \to 0} \frac{x^2}{2e^x - 2\cos x - \operatorname{sen} x}$$

USA $+1 \in -1$