

VALORE ASSOLUTO

È IL NUMERO PRESO SENZA SEGNO.

$$|+3| = 3$$

$$|-3| = 3$$

$$|+\frac{1}{2}| = \frac{1}{2}$$

$$|-\frac{1}{2}| = \frac{1}{2}$$

$$|+\sqrt{2}| = \sqrt{2}$$

$$|-\sqrt{2}| = \sqrt{2}$$

$$|0| = 0$$

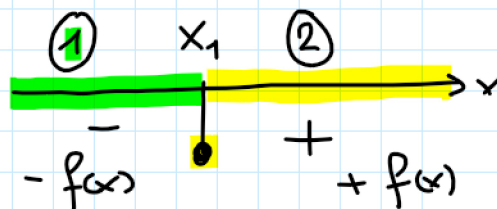
IL SUO CONTENUTO SI CHIAMA ARGOMENTO

$$|f(x)| = \begin{cases} f(x) & \text{se } f(x) \geq 0 \\ -f(x) & \text{se } f(x) < 0 \end{cases}$$

COME SCIOLGO IL MODULO?

MI OCCORRE CONOSCERE IL SEGNO DELL' ARGOMENTO

RISOLVO $f(x) \geq 0$

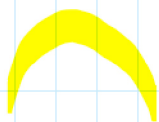


$$|f(x)| = \begin{cases} -f(x) & \text{se } x < x_1 \\ +f(x) & \text{se } x \geq x_1 \end{cases}$$

ESERCIZIO

$$|1 - x^2|$$

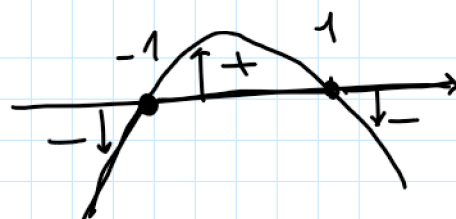
$$1 - x^2 \geq 0$$



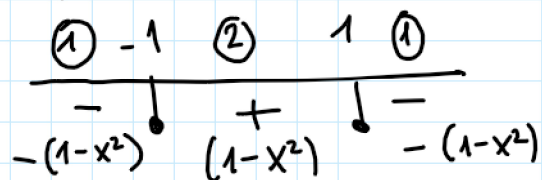
$$\text{REA } 1 - x^2 = 0$$

$$x = \pm 1$$

PARABOLA



$$|1 - x^2| = \begin{cases} (1) & -(1 - x^2) \text{ se } x < -1 \vee x > 1 \\ (2) & +(1 - x^2) \text{ se } -1 \leq x \leq 1 \end{cases}$$



CONSIDERAZIONI

$$|f(x)| \begin{cases} + \forall x \in \mathbb{R}, f(x) \neq 0 \\ 0 \rightarrow f(x) = 0 \end{cases}$$

$$(f(x))^2 \begin{cases} \rightarrow + \forall x \in \mathbb{R}, f(x) \neq 0 \\ \rightarrow 0 \quad f(x) = 0 \end{cases}$$

EQUAZIONI

$$1) |x - 5| = -4 \quad \text{IMP}$$

$$(x - 5)^2 = -4 \quad \text{IMP}$$

$$2) |x - 5| = 0 \quad \begin{matrix} x - 5 = 0 \\ x = 5 \end{matrix}$$

$$(x - 5)^2 = 0 \quad \begin{matrix} x - 5 = 0 \\ x = 5 \end{matrix}$$

$$3) \overbrace{|x - 5|}^{\substack{9 \\ \uparrow}} = 9$$

$$\overbrace{(x - 5)^2}^{\substack{\sqrt{9} \\ \uparrow}} = 9$$

$$x - 5 = -9 \quad \vee \quad x - 5 = +9$$

$$x = -4 \quad \vee \quad x = 14$$

$$x - 5 = -\sqrt{9} \quad \vee \quad x - 5 = +\sqrt{9}$$

$$x = 2$$

$$\vee x = 8$$

$$\begin{array}{c} \text{UGUALI} \\ |1-x| = |2x-3| \\ \text{OPPOSTI} \end{array}$$

$$\rightarrow 1-x = 2x-3$$

$\vee$

$$1-x = -(2x-3)$$

$$-3x = -4$$

$$x = \frac{4}{3}$$

$\vee$

$$1-x = -2x+3$$

$$x = 2$$

RIASSUNTO

$$|f(x)| = k \quad k \in \mathbb{R}$$

$$k < 0$$

IMP.

$$k = 0$$

$\downarrow$

$$f(x) = 0$$

$$k > 0$$

$\downarrow$

$$f(x) = -k \vee f(x) = +k$$

$$|f(x)| = |g(x)|$$

$$f(x) = g(x)$$

$\vee$

$$f(x) = -g(x)$$

PER VOI

$$1) \quad |x-1| = 6 \quad \text{Sol} \quad x = -5 \quad x = 7$$

$$2) \quad |x^2 - 1| = -5 \quad \text{NON VE LO DRÒ}$$

$$3) \quad |x^2 - 1| = 8 \quad \text{Sol} \quad x = +3 \quad x = -3$$

$$4) \quad |1 - |x|| = \frac{1}{2} \quad \text{Sol} \quad x = \pm \frac{1}{2} \quad x = \pm \frac{3}{2}$$

SUGGERIMENTO:
TOGLI PRIMA IL
MODULO ESTERNO

$$5) \quad |x^2 - 3x| = |x^2 + x| \quad \text{Sol} \quad x = 0 \quad x = 1$$

DISEQUAZIONI

VA TENUTO PRESENTE

$$\text{CHE} \quad |f(x)| \begin{cases} > \forall x \in \mathbb{R} \text{ con } f(x) \neq 0 \\ = 0 \text{ se } f(x) = 0 \end{cases}$$

1° TIPO : AL 2° MEMBRO HO UN NUMERO NEGATIVO

$$|x-1| < -4$$

IMP

$$|x-1| \leq -5$$

IMP

$$|x-1| > -9$$

$\forall x \in \mathbb{R}$

$$|x-1| \geq -3$$

$\forall x \in \mathbb{R}$

2° TIPO : AL 2° MEMBRO HO 0

(È IL CASO PIÙ BRUTTO)

$$\left| x - 5 \right| \begin{matrix} + \text{OK} \\ > 0 \\ 0 \text{ KO} \end{matrix} \quad \left(\begin{matrix} \forall x \in \mathbb{R} \\ x \neq 5 \end{matrix} \right) \quad \left| x - 5 \right| \begin{matrix} + \text{OK} \\ \geq 0 \\ = 0 \text{ OK} \end{matrix} \quad \left(\forall x \in \mathbb{R} \right)$$

$x - 5 \neq 0$
 $x \neq 5$

$$\left| x - 5 \right| \begin{matrix} + \text{KO} \\ < 0 \\ 0 \text{ KO} \end{matrix} \quad \text{IMP} \quad \nexists x \in \mathbb{R}$$

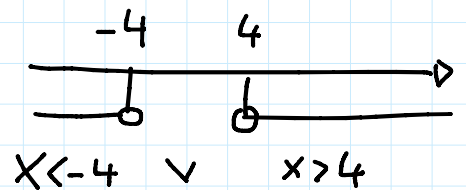
$$\left| x - 5 \right| \begin{matrix} + \text{KO} \\ \leq 0 \\ 0 \text{ OK} \end{matrix} \quad \begin{matrix} x - 5 = 0 \\ x = 5 \end{matrix} \quad (x = 5)$$

$$\left(x - 5 \right)^2 \begin{matrix} + \text{OK} \\ \geq 0 \\ 0 \text{ OK} \end{matrix} \quad \forall x \in \mathbb{R}$$

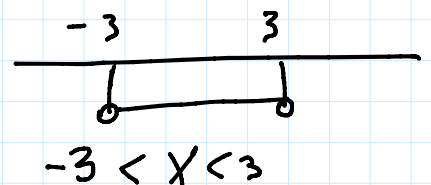
$$\left(x - 5 \right)^2 \begin{matrix} + \text{OK} \\ > 0 \\ 0 \text{ KO} \end{matrix} \quad \begin{matrix} \forall x \in \mathbb{R} \\ x \neq 5 \end{matrix}$$

3° TIPO: AL 2° MEMBRO HO UN NUMERO POSITIVO

ESEMPIO $|x| > 4$



$$|x| < 3$$

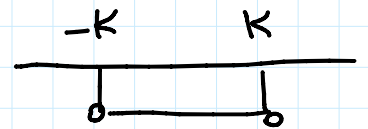


$$|f(x)| > K \quad (K \in \mathbb{R}^+)$$



$$f(x) < -K \vee f(x) > K$$

$$|f(x)| < K \quad (K \in \mathbb{R}^+)$$



$$-K < f(x) < K$$

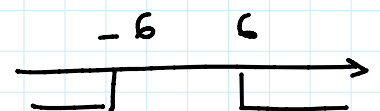
COMPRESO < <

↓
SISTEMA "GIF"

$$\begin{cases} f(x) < K \\ f(x) > -K \end{cases}$$

ESERCIZI

$$|x^2 - 5x + 6| > 6$$



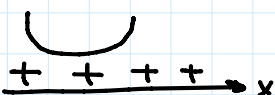
✓ 2 ✓ ✓ ✓ 2 ✓ ✓ ✓

$$x^2 - 5x + 6 < -6 \quad \checkmark$$

$$x^2 - 5x + 12 < 0$$

1) REA $x^2 - 5x + 12 = 0$

$$\Delta = +25 - 48 < 0 \quad x_{1,2} \notin \mathbb{R}$$

2) PARABOLA 

Non ci sono
i - $\Rightarrow$ sol IMP $\nexists x \in \mathbb{R}$

$$x^2 - 5x + 6 > 6$$

$$x^2 - 5x > 0$$

1) REA $x^2 - 5x = 0$

$$x(x-5) = 0$$

$$\downarrow \quad \downarrow$$

$$x=0 \quad x-5=0$$

$$x=5$$

2) PARABOLA



$$x < 0 \vee x > 5$$

$$|1 - 2x| < 1$$

$$-1 < 1 - 2x < 1$$

$$-2 < -2x < 0$$

$$-1 < -x < 0$$

CAMBIO
SEGNO A TUTTI E VERSO

$$1 > x > 0 \quad \text{RIORDINO}$$

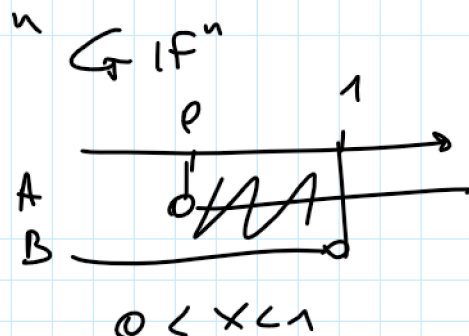
$$0 < x < 1$$



OPPURE

$$-1 < 1 - 2x < 1$$

$$\begin{array}{l} A \left\{ \begin{array}{l} 1 - 2x < 1 \\ 1 - 2x > -1 \end{array} \right. \quad A \quad -2x < 0 \quad 2x > 0 \quad x > 0 \\ B \left\{ \begin{array}{l} 1 - 2x < 1 \\ 1 - 2x > -1 \end{array} \right. \quad B \quad -2x > -2 \quad 2x < 2 \quad x < 1 \end{array}$$



OPPURE

$$\left(|1 - 2x| \right)^2 < (1)^2$$

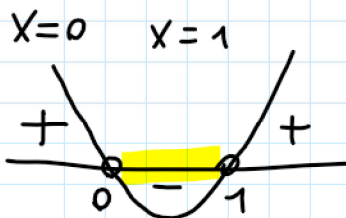
$$1 - 4x + 4x^2 < 1$$

$$4x^2 - 4x < 0 \quad (:4)$$

$$x^2 - x < 0$$

REA $x^2 - x = 0 \quad x(x-1) = 0$

$\downarrow \quad \downarrow$

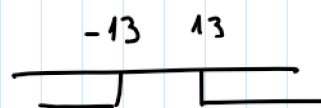


PARABOLA

$$0 < x < 1$$

$$|3x^2 + 1| > 13$$

$$3x^2 + 1 < -13$$



$$3x^2 + 1 > 13$$

$$3x^2 + 14 < 0$$

$$\nexists x \in \mathbb{R} \text{ IMP}$$

$$3x^2 - 12 \geq 0 \quad (:3)$$

$$x^2 - 4 \geq 0 \quad \frac{+}{-2} \quad \frac{-}{+2}$$

$$x < -2 \vee x > 2$$

PER VOI

$$|4x^2 + 1| < 2$$

$$\text{sol} \quad -\frac{1}{2} < x < \frac{1}{2}$$

$$\left| \frac{x+3}{x-2} \right| < 4$$

$$\text{sol} \quad x < 1 \vee x > \frac{11}{3}$$

$$|x^2 + 2x| > 3$$

$$\text{sol} \quad x < -3 \vee x > 1$$

$$\frac{|x| - 1}{x + 3} \geq 0$$

$$\text{sol} \quad -3 < x \leq -1 \vee x \geq 1$$

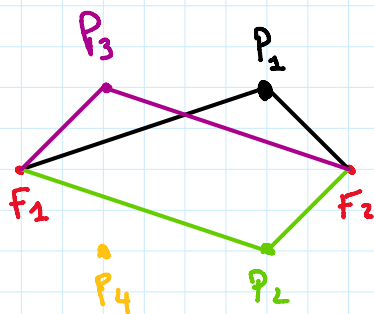
SUGGERIMENTO. PARTI SUBITO
CON LA DISEQUAZIONE FRATTA

$$N \geq 0 \quad \dots$$

$$D > 0 \quad \dots$$

ELLISSE

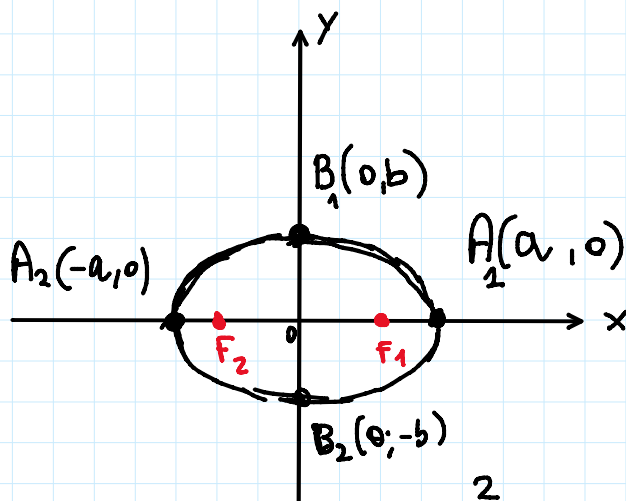
DATI 2 PUNTI DETTI FUOCHI L'ELLISSE
È L'INSIEME DEI PUNTI DEL PIANO PER I
QUALI È COSTANTE LA SOMMA DELLE DISTANZE
DAI FUOCHI



$$\overline{P_1 F_1} + \overline{P_2 F_2}$$

$$\overline{P_2 F_1} + \overline{P_2 F_2}$$

$$\overline{P_3 F_1} + \overline{P_3 F_2}$$



EQUAZIONE CANONICA

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$A_1(a, 0)$ SODDISFA L'ENUNCIATO!

$A_2(-a, 0)$

$$\downarrow$$

$$\frac{(-a)^2}{a^2} + \frac{0}{b^2} = 1$$

$$1 = 1 \quad \text{SI}$$

$$\frac{a^2}{a^2} + \frac{0^2}{b^2} = 1$$

$$1 = 1 \quad \text{SI}$$

$B_1(0, b)$ $B_2(0, -b)$ SODDISFANO L'ENUNCIATO

$A_{1,2}(\pm a; 0)$

$B_{1,2} = (0; \pm b)$

VERTICI

$$4x^2 + 2y^2 = 16$$

$$x^2 + y^2 = 1$$

$$4x^2 + 2y^2 = 16$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ROMPE

↓
DIVIDIDO POR 16

$$\frac{1}{\cancel{16}} \frac{x^2}{\cancel{4}} + \frac{1}{\cancel{2}} \frac{y^2}{\cancel{16}} = 1 \quad \text{OK}$$

4 8

$$\frac{x^2}{4} + \frac{y^2}{8} = 1$$

$$a^2 = 4$$

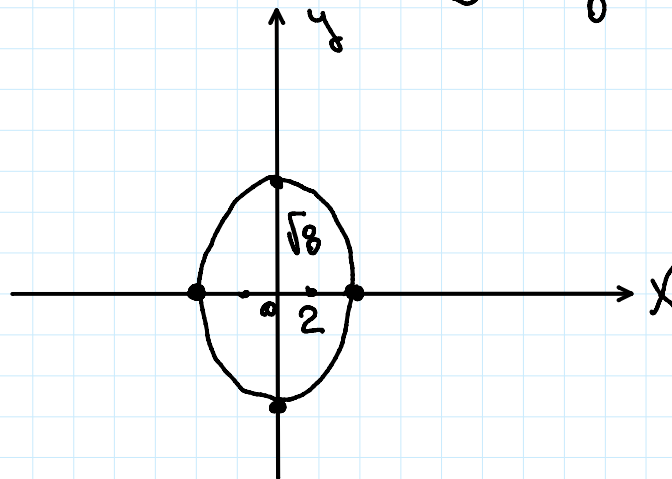
$$b^2 = 8$$

$$a = 2$$

$$b = \sqrt{8} = \sqrt{4 \cdot 2}$$

$$= \sqrt{4} \cdot \sqrt{2}$$

$$= 2\sqrt{2}$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a > b$$

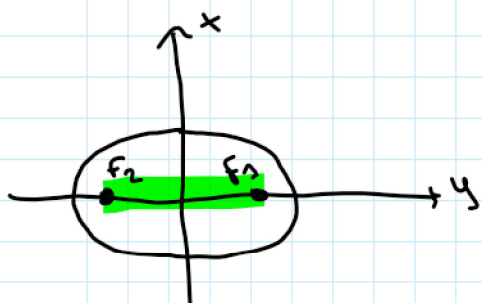
$F_{1,2} \in \text{axe } x$

$$a < b$$

$F_{1,2} \in \text{axe } y$

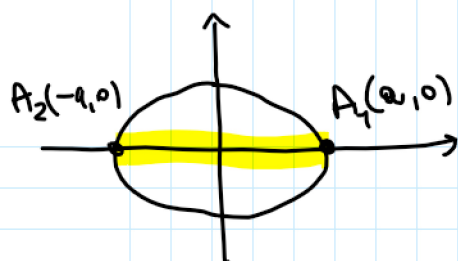
$$F_{1,2} (\pm c, 0)$$

$$c = \sqrt{a^2 - b^2}$$



DISTANZA FOCAL

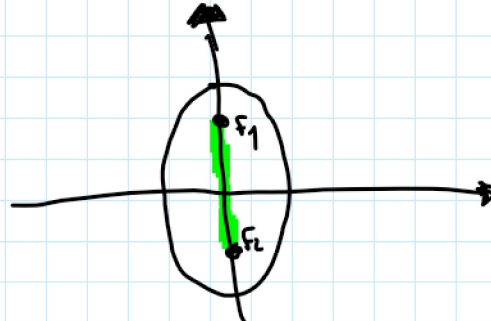
$$2c$$



ASSE MAGGIORE 2a

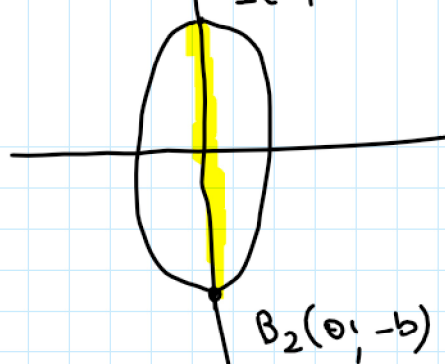
$$F_{1,2} (0; \pm c)$$

$$c = \sqrt{b^2 - a^2}$$



DISTANZA FOCAL 2c

$$B_1(0, b)$$



ASSE MAGGIORE 2b

ECCENTRICITA' : e

$$e = \frac{\text{DISTANZA FOCAL}}{\text{ASSE MAGGIORE}}$$

$$e = \frac{2c}{2a}$$

$$e = \frac{c}{a}$$

$$e = \frac{2c}{2b}$$

$$e = \frac{c}{b}$$

L'ECCENTRICITA' FORNISCE LO SCHIACCIAMENTO DELL'ELLISSE

$$e = \frac{c}{a} \quad c = \sqrt{a^2 - b^2}$$

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

$$e = \frac{\sqrt{a^2 - b^2}}{\sqrt{a^2}}$$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$e = \sqrt{\frac{a^2}{a^2} - \frac{b^2}{a^2}}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \frac{c}{b} \quad c = \sqrt{b^2 - a^2}$$

$$e = \frac{\sqrt{b^2 - a^2}}{b}$$

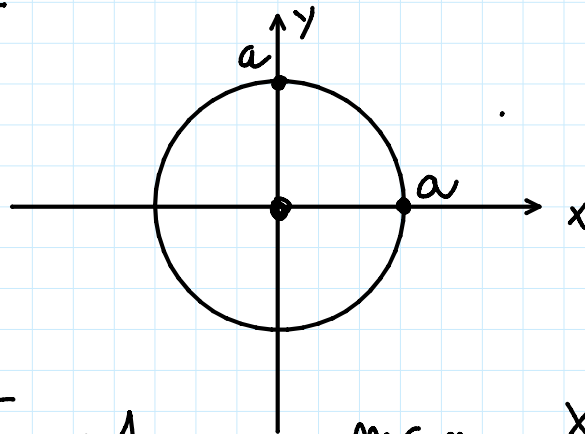
$$e = \frac{\sqrt{b^2 - a^2}}{\sqrt{b^2}}$$

$$e = \sqrt{\frac{b^2 - a^2}{b^2}}$$

$$e = \sqrt{\frac{b^2}{b^2} - \frac{a^2}{b^2}}$$

$$e = \sqrt{1 - \frac{a^2}{b^2}}$$

SE $a = b$?



$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$$

m.c.m. $x^2 + y^2 = a^2$
CIRCONFERENZA
CENTRO $O(0,0)$
RAGGIO a

$$\text{SE } a = b \quad e = \sqrt{1 - \frac{a^2}{b^2}}$$

$$e = \sqrt{1-1} = 0$$