LEZ F

lunedì 9 settembre 2024 15:28

PROPRIETÀ LOGARITMI

STESSA BASE

$$\log_a b + \log_a c = \log_a (b \cdot c)$$

$$\log_a b - \log_a c = \log_a \left(\frac{b}{c}\right)$$

PROPRIETÀ ESPONENTE DELL'ARGOMENTO

$$\log_a b^m = m \cdot \log_a b$$

ATTENTI

$$\log_a^N b \neq \log_a b^N$$

$$\downarrow$$

$$(\log_a b)^N \neq \log_a (b)^N$$

CAMBIO BASE

$$\log_a b = \frac{\log_c b}{\log_c a}$$

nuova

$\log_a b$ = $\frac{\log_c b}{\log_c a}$

Nuova base la chiamo c

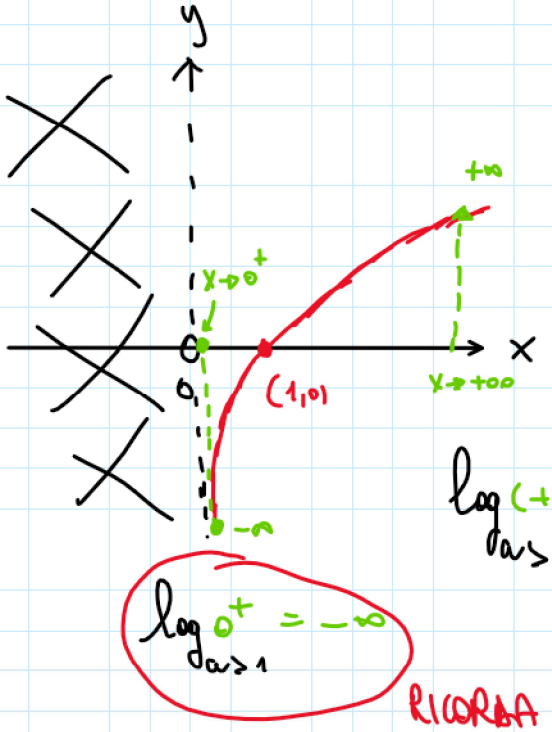
ESERCIZIO $\log_4 x$ LO VOGLIO CAMBIARE IN $\log_2$

$$\log_4 x = \frac{\log_2 x}{\log_2 4} = \frac{\log_2 x}{2} = \frac{1}{2}\log_2 x$$

GRAFICI

$y = \log_a x$ DOMINIO $x > 0$

$a > 1$

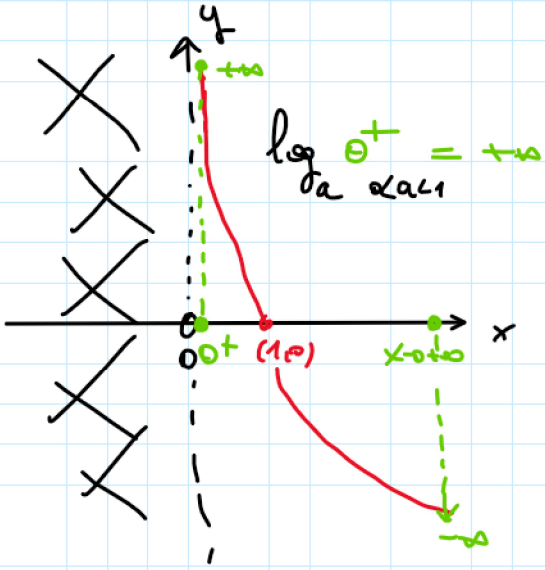


$\log_{a>1}(+\infty) = +\infty$

$\log_{a>1} 0^+ = -\infty$

RICORDA BENE

$0 < a < 1$



$\log_{a<a<1} 0^+ = +\infty$

$\log_{a<a<1} (+\infty) = -\infty$

RISOLVI

$3^x = 5$

$$\ln 3^{x} = \ln 5$$

$$x \ln 3 = \ln 5 \qquad (:\ln 3) \qquad \frac{x \ln 3}{\ln 3} = \frac{\ln 5}{\ln 3}$$

numeri reali

$$x = \frac{\ln 5}{\ln 3} \Rightarrow x = \frac{1,61}{1,09} \qquad x = 1,48$$

$$3^{x} = 5$$

$$\log 3^{x} = \log 5$$

$$x \log 3 = \log 5 \qquad x = \frac{\log 5}{\log 3} \Rightarrow x = \frac{0,70}{0,47} = 1,48$$

$$3^{x} = 5$$

$$\log_3 3^{x} = \log_3 5$$

$$x \cdot \log_3 3 = \log_3 5 \qquad (\log_3 3 = 1)$$

$$x = \log_3 5 \qquad x = 1,48$$

CAMBIO BASE DA $\log_3$ A $\ln$

CAMBIO BASE DA $\log_3$ A $\log$

$$\frac{\ln 5}{\ln 3} = \log_3 5 = \frac{\log 5}{\log 3}$$

$\log 3$ | 1,48

$\log 3$ | 1,48

$\log_2 3$ | 1,48

EQUAZIONI LOGARITMICHE

SUBITO

FACCIO IL DOMINIO : ARGOMENTO > 0

SE HO PIÙ DI UN DOMINIO

$\begin{cases} \text{ARGOM. } 1 > 0 \\ \text{ARGOM. } 2 > 0 \\ \dots \end{cases}$ "GIF"

1° TIPO USO LE PROPRIETÀ E GIUNGO A

$$\log_a x_1 = \log_a x_2 \xrightarrow{\text{INIETTIVA}} x_1 = x_2$$

TROVO LA SOL $x = \dots$ E CONTROLLO

SE $x \in$ DOMINIO (SI)

SE $x \notin$ DOMINIO (NO)

2° TIPO HO LA FORMA

$$\log_a x_1 = c \xrightarrow{\text{DEF}} x_1 = a^c$$

TROVO LA SOL $x = \dots$ E CONTROLLO

SE $x \in$ DOMINIO (SI)

SE $x \notin$ DOMINIO (NO)

3° TIPO I VARI LOGARITMI HA L'ESPONENTE

$\log_2^3 x$ o $\ln^2 x$ ...

PONGO E DEPONGO

TROVO LA SOL $x = \ldots$ E CONTROLLO

se $x \in$ DOMINIO (SI)

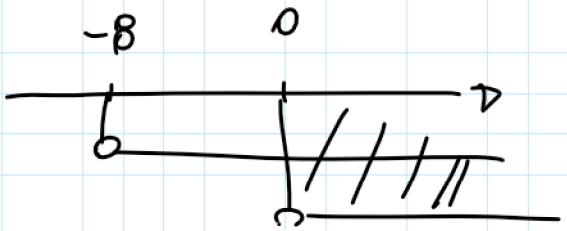
se $x \notin$ DOMINIO (NO)

ES

$$\log_{13}(x+8) = 2\log_{13} 3 - \log_{13} x$$

DOMINIO

$$D \begin{cases} x+8 > 0 \\ x > 0 \end{cases} \quad \begin{cases} x > -8 \\ x > 0 \end{cases}$$



$D \quad x > 0$

$$\log_{13}(x+8) = 2\log_{13} 3 - \log_{13} x$$

$$\log_{13}(x+8) = \log_{13} 9 - \log_{13} x$$

$$\log_a\left(\frac{b}{c}\right)$$

$$\log_{13}(x+8) = \log_{13}\left(\frac{9}{x}\right)$$

$$x+8 = \frac{9}{x} \qquad \text{m.c.m}$$

$$\frac{x^2+8x}{x} = \frac{9}{x} \qquad \text{C.E } x \neq 0$$

$x^2 + 8x - 9 = 0$

$x_1 = 1$ (SI)

$x_2 = -9$ (NO)

$\left(x_2 = \frac{c}{a}\right)$

P

$$\log_2 x + 3\log_2 x = 10$$

DOMINIO $x > 0$

$$\log_2 x + \log_2 x^3 = 10$$

$$\log_2 (x \cdot x^3) = 10$$

$$\log_2 x^4 = 10$$

DEF $\log_a b = c \Rightarrow b = a^c$

$x^4 = 2^{10}$ USO $\pm \sqrt[4]{\ }$

$\sqrt[4]{x^4} = \pm \sqrt[4]{2^{10}}$

$x = \pm \sqrt[\cancel{4}\,2]{2^{\cancel{10}\,5}}$

$x_1 = -\sqrt{2^5}$ NO $\quad x_2 = +\sqrt{2^5}$ SI

?D

"PONGO" e "DEPONGO"

$\log_3^{(2)} x + 2\log_3 x - 3 = 0$

DOMINIO $\quad x > 0$

$\log_3 x = t$ "TRUCCO"

$t^2 + 2t - 3 = 0$

$t_1 = 1$

$t_2 = -3$

$\log_3 x = 1 \Rightarrow x = 3^1 \quad x = 3$ (SI)

$\log_3 x = -3 \Rightarrow x = 3^{-3} \quad x = \frac{1}{27}$ (SI)

DISEQUAZIONI LOGARITMICHE

A Dominio 1

B Dominio 2 "GIF"

... .

.. ESERCIZIO

ESERCIZIO

1° TIPO | 2° TIPO | 3° TIPO

COME LE EQUAZIONI

↓

MA ATTENTI AL "a"

MA ATTENTI AD "a"

$a>1$ CRESCE — NON CAMBIO VERSO

$0<a<1$ DECRESCE — CAMBIO VERSO

$$\ln x \leq -1$$

DOM. $\begin{cases} x>0 \\ \ln x \leq -1 \end{cases}$ ESERC.

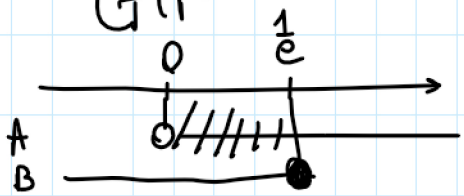
A $x>0$

B $\ln x \leq -1$ $\quad a>1$ ($a=e$ 47)

$x \leq e^{-1}$

$x \leq \frac{1}{e}$

GIF



$0<x\leq \frac{1}{e}$

ϕ

$$\ln(1-x^2)-1<0$$

D $\begin{cases} 1-x^2>0 & A \\ \ln(1-x^2)-1<0 & B \end{cases}$ ESER.

A $1-x^2>0$ REA

RISOLVO EQUAZIONE ASSOCIATA

① $1-x^2=0$

$-x^2=-1$

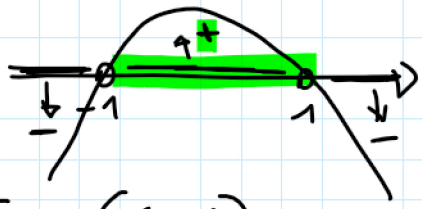
$x^2=1$

$\sqrt{x^2}=\pm\sqrt{1}$

$x^2 = \pm\sqrt{1}$

$x = \pm 1$

PARABOLA



$A \left[\begin{array}{l} (-1;1) \\ \text{o} \\ -1 < x < 1 \\ \text{o} \\ ]-1;1[\end{array} \right.$

B $\ln(1-x^2) - 1 < 0$

$\ln(1-x^2) < 1$

↓ LO VOGLIO TRASFORMARE IN LOGARITMO IN BASE e

$\ln(1-x^2) < \ln e^1$ $\quad a > 1 \quad e = 2{,}7$

$1 - x^2 < e$

2° GRADO: SPOSTO TUTTO AL 1° MEMBRO E LASCIO 0 AL 2°

$1 - x^2 - e < 0$

REA $1 - x^2 - e = 0$

$-x^2 = e - 1$

↑ CAMBIO SEGNO

x^2

$$x^2 = 1 - e \qquad \text{METTO } \pm\sqrt{}$$

$$\sqrt{x^2} = \pm\sqrt{1-e} \qquad x_{1,2} = \pm\sqrt{1-e}$$

$x_{1,2} \in \mathbb{R}$

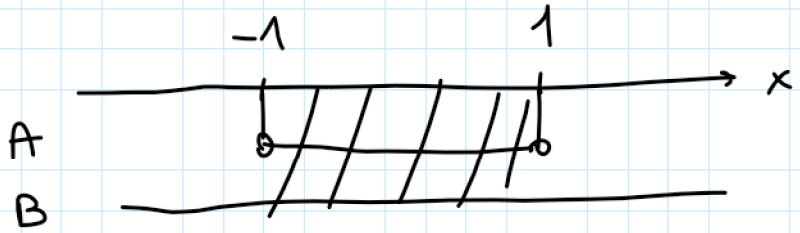
PARABOLA



$\forall x \in \mathbb{R}$

$(-\infty; +\infty)$

$]-\infty; +\infty[$



$-1 < x < 1$

ϕ

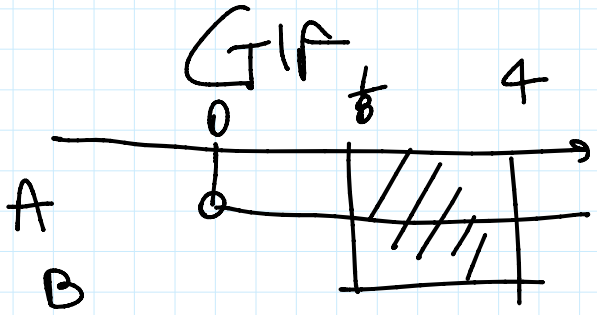
$$-2 < \log_{\frac{1}{2}} x < 3$$

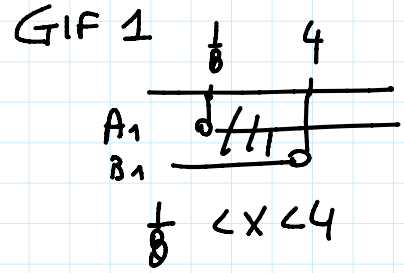
$$\begin{cases} x > 0 \\ -2 < \log_{\frac{1}{2}} x < 3 \end{cases} \qquad \begin{matrix} A & x > 0 \\ B & -2 < \log_{\frac{1}{2}} x < 3 \end{matrix} \qquad \text{COMPRESO}$$

$$A_1 \begin{cases} \log_{\frac{1}{2}} x \geq 3 \\ B_1 \quad \log_{\frac{1}{2}} x \leq -2 \end{cases}$$

$A_1 \quad 0 < a < 1$

$x \geq \left(\frac{1}{2}\right)^3 \quad x \geq \frac{1}{8}$





$$\frac{1}{8} < x < 4$$

OPPURE

DOMINIO $\begin{cases} x \geq 0 \\ \log_{\frac{1}{2}} x < 3 \\ \log_{\frac{1}{2}} x > -2 \end{cases}$

DISEQUAZIONI 2° GRADO

1) VANNO MESSE NELLA FORMA

ax^2+bx+c COMPLETA	≥ 0	≤ 0	> 0	< 0
	PALLE PIENE		PALLE VUOTE	
ax^2+bx INCOMPLETA	≥ 0	≤ 0	> 0	< 0
ax^2+c INCOMPLETA	≥ 0	≤ 0	> 0	< 0

COMANDA "a": NUMERO DI x^2 "

a+ ∪

a− ∩

2) SCRIVI REA (RISOLVO L'EQUAZIONE ASSOCIATA) E LE PONI $=0$ (TUTTE: COMPLETA INCOMPLETA)

$ax^2+bx+c=0$ FORMULA

$ax^2+bx=0$ RACCOLGO x

$ax^2+c=0$ USO $\pm\sqrt{}$

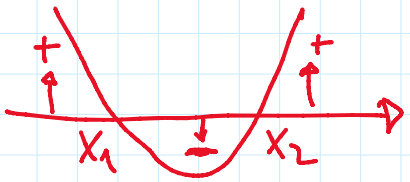
E TROVI X_1 e X_2 $\in \mathbb{R}$ $\notin \mathbb{R}$ LI BUTTO

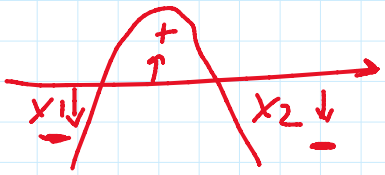
3) DISEGNI LA PARABOLA IN BASE AD a (PRIMA DI REA) E IN BASE AD X_1 e X_2

PARABOLA

HO 2 VALORI DISTINTI E REALI DI X_1 e X_2

$a>0$

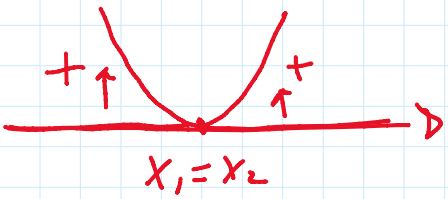


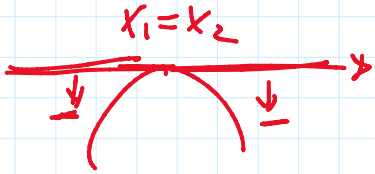


$a<0$

HO 2 VALORI COINCIDENTI E REALI $X_1=X_2$

$a>0$



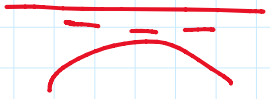


$a<0$

HO 2 VALORI <u>COMPLESSI</u> X_1 e $X_2 \notin \mathbb{R}$

$a>$





$a<0$

GUARDO IL VERSO DELLA DISEQUAZ.

PRIMA DI REA:

≥ 0 > 0 PRENDO + ≤ 0 < 0 PRENDO −

$\log_3(x^2-1) > 0$ ⟶ A / B ⟶ GIF

A $x^2-1>0$

B $\log_3(x^2-1)>0$

A $x^2-1>0$ REA $x^2-1=0$ $x^2=1$ $x=\pm 1$

PARABOLA



A $x<-1 \vee x>1$

B $\log_3(x^2-1) \geq 0$ ↓ $\log_3$

$\log_3(x^2-1) > \log_3 3^0$ $a>1$

$x^2-1>3^0$

$x^2-1>1$ $x^2-2>0$

REA $x^2-2=0$

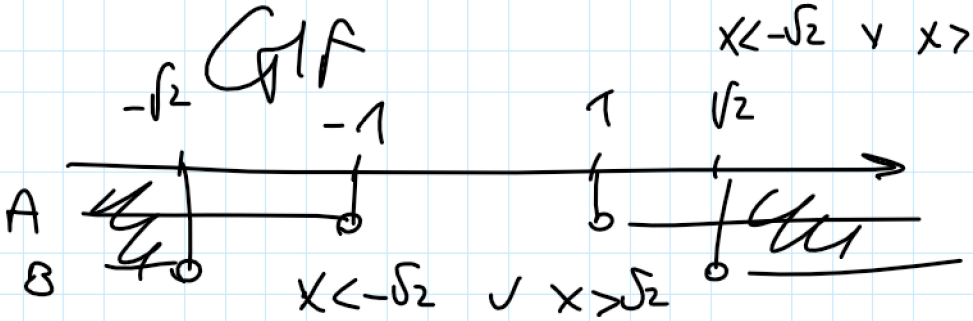
$x^2=2$ $x=\pm\sqrt{2}$

PAR



$x<-\sqrt{2} \vee x>\sqrt{2}$

GIF



$x<-\sqrt{2} \vee x>\sqrt{2}$

$$\log_{\frac{1}{2}}(x^2-x) > \log_{\frac{1}{2}}(x^2+1)$$

φ

A $x^2-x>0$

B $x^2+1>0$

C $\log_{\frac{1}{2}}(x^2-x) > \log_{\frac{1}{2}}(x^2+1)$

A $x^2-x>0$ REA $x^2-x=0$

$x(x-1)=0$

$x=0$

$x=1$



$x<0 \vee x>1$

B $x^2+1>0$

È LA SOMMA DI 2 QUANTITÀ POSITIVE x^2 (POSITIVO O ZERO)

1 (POSITIVO)

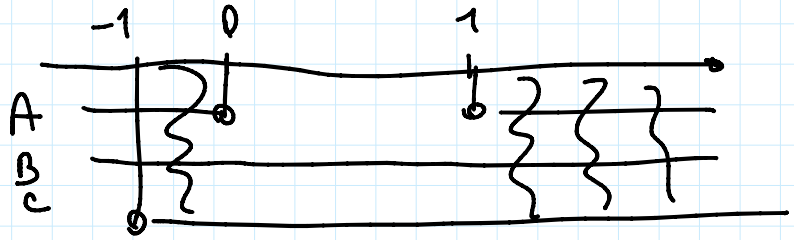
$x^2+1>0 \quad \forall x \in \mathbb{R}$

C $\log_{\frac{1}{2}}(x^2-x) > \log_{\frac{1}{2}}(x^2+1)$ $\quad 0<a<1$

$x^2-x < x^2+1$

$-x<1 \rightarrow x>-1$

GIF



$-1<x<0 \quad \vee \quad x>1$

$\log_2^2 x - \log_2 x > 0$ SOL $0<x<1 \vee x \geq 2$

$\ln^2 x + \ln x \leq 0$ SOL $\frac{1}{e} \leq x \leq 1$