LEZ E

venerdì 6 settembre 2024 09:08

PROPRIETÀ POTENZE

LE USO SOLO CON MOLTIPLIC. E DIVIS.

STESSA BASE

$$a^n \cdot a^m = a^{n+m}$$

$$a^{n+m} = a^n \cdot a^m$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$a^{n-m} = \frac{a^n}{a^m}$$

STESSO ESPON

$$a^n \cdot b^n = (a \cdot b)^n$$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

POTENZA DI POTENZA

$$(a^n)^m = a^{n \cdot m}$$

$$a^{n \cdot m} = (a^n)^m = (a^m)^n$$

ESPONENTE NEGATIVO

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{1}{a^n} = a^{-n}$$

ESPONENTE ZERO

$$a^0 = 1 \quad \text{TRANNE} \quad 0^0 \quad \text{F. I.}$$

LA RADICE

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

? $\sqrt{-9}$ $\nexists$ IN $\mathbb{R}$

$3i \rightarrow (3i)^2 = 9i^2 = -9$ ($i^2 = -1$)

? $(-4)^{\frac{1}{2}}$ $\nexists$ IN $\mathbb{R}$

$2i$ $(-4)^{\frac{1}{2}} = \sqrt{-4}$

$(-4)^{\frac{1}{2}} = \sqrt{-4}$

FUNZIONE ESPONENZIALE

$f : \mathbb{R} \longrightarrow \mathbb{R}$

$D \subseteq \mathbb{R}$ $C \subseteq \mathbb{R}$

$f(x) = a^x$ $y = a^x$ $a \in \mathbb{R}^+ \setminus \{1\}$

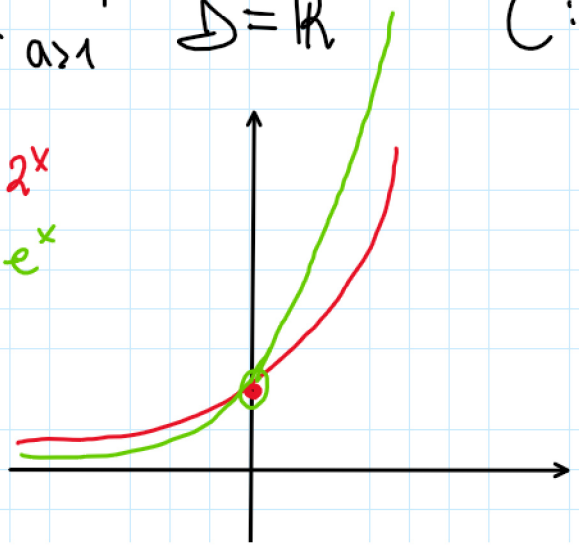
$f : \mathbb{R} \longrightarrow \mathbb{R}$

$D = \mathbb{R}$ $C : \mathbb{R}^+$

$y = a^x$ $a > 1$

$y = 2^x$

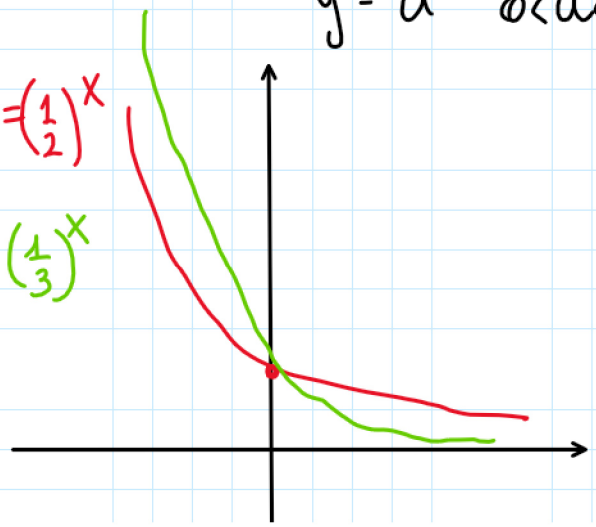
$y = e^x$



$y = a^x$ $0 < a < 1$

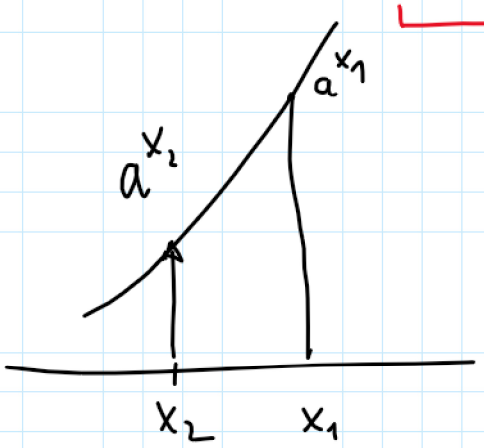
$y = (\frac{1}{2})^x$

$y = (\frac{1}{3})^x$



1) $C: \mathbb{R}^+ \Rightarrow a^x > 0 \quad \forall x \in \mathbb{R}$

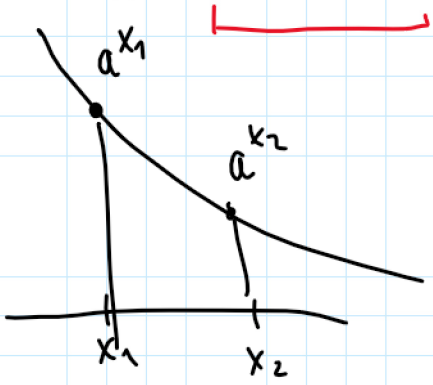
2) CRESCENTE $\Rightarrow a^{x_1} > a^{x_2} \Rightarrow x_1 > x_2$



1) $C: \mathbb{R}^+ \Rightarrow a^x > 0 \quad \forall x \in \mathbb{R}$

2) DECRESCENTE $\Rightarrow$

$a^{x_1} > a^{x_2} \Rightarrow x_1 < x_2$



EQUAZIONI ESPONENZIALI

LA FUNZIONE ESPONENZIALE È INIETTIVA

PER CUI SE SO CHE $f(x_1) = f(x_2)$

DEDUCO CHE ALLORA $x_1 = x_2$

$$a^{x_1} = a^{x_2} \Rightarrow x_1 = x_2$$

1° TIPO: CERCO DI USARE LE PROPRIETÀ

E DI GIUNGERE ALLA FORMA

$$a^{x_1} = a^{x_2}$$

IN QUESTO MODO SARÀ $x_1 = x_2$

IN GENERE SI USA QUESTA PROCEDURA

QUANDO TRA GLI ESPONENZIALI HO

MOLTIPLICAZIONI E/O DIVISIONI.

$$8 \cdot 4^{x+3} = \frac{1}{32}$$

$$2^3 \cdot \left(2^2\right)^{(x+3)} = \frac{1}{2^5}$$

$$2^3 \cdot 2^{2x+6} = 2^{-5}$$

$$2^{3+2x+6} = 2^{-5} \qquad \left(a^{x_1} = a^{x_2} \Rightarrow x_1 = x_2\right)$$

$$3 + 2x + 6 = -5$$

$$2x = -5 - 9$$

$$2x = -14$$

$$\boxed{x = -7}$$

$p(-7)$ V/F

$$8 \cdot 4^{-7+3} = \frac{1}{32}$$

$$8 \cdot 4^{-4} = \frac{1}{32}$$

$$8 \cdot \frac{1}{4^4} = \frac{1}{32}$$

$$\cancel{8}^{1} \cdot \frac{1}{\cancel{256}^{32}} = \frac{1}{32}$$

$p(-7) \Rightarrow V.$

2° TIPO $a^{x_1} = k \quad (k \in \mathbb{R})$

2° TIPO $a = k$ $(k \in \mathbb{R})$

- $2^x = -3$ IMP
  FALSO $\forall x \in \mathbb{R}$
  PERCHÈ $a^x > 0 \ \forall x \in \mathbb{R}$
- $5^x = 0$ IMP
  FALSO $\forall x \in \mathbb{R}$
- $7^x = 49$
  $7^x = 7^2$
  $x = 2$

$7^x = 3$

↓ USO IL LOGARITMO

3° TIPO : TRA GLI ESPONENZIALI HO ADDIZIONI / SOTTRAZIONI

"PONGO E DEPONGO"

MA PRIMA LIBERO GLI ESPONENTI DA ADDIZIONI E SOTTRAZIONI USANDO $a^{n+m} = a^n \cdot a^m$

$a^{n-m} = \frac{a^n}{a^m}$

$2^{x+1} \oplus 2^{x+3} = 80$

↓ PONGO E DEPONGO

$2^x \cdot 2^1 + 2^x \cdot 2^3 = 80$

PONGO $2^x = t$

$t \cdot 2 + t \cdot 8 = 80$

$2t + 8t = 80$

$10t = 80$

$t = 8$

DEPONGO $2^x = t$ ← $2^x = 8 \quad x = 3$

$4^x - 6 \cdot 2^x + 8 = 0$

$\downarrow$

$(2^2)^x - 6 \cdot 2^x + 8 = 0$

$\downarrow$

$(2^x)^2 - 6 \cdot 2^x + 8 = 0 \qquad 2^x = t$

$t^2 - 6t + 8 = 0$

$(t-2)(t-4) = 0$ < $t=2 \rightarrow 2^x = 2 \quad x=1$; $t=4 \rightarrow 2^x = 4 \quad x=2$

OPPURE

$\Delta = (-6)^2 - 4(1)(8) = 36 - 32 = 4$

$t_{1,2} = \frac{6 \pm 2}{2}$ < $t_1 = \frac{6-2}{2} = \frac{4}{2} = 2$; $t_2 = \frac{6+2}{2} = \frac{8}{2} = 4$

ESERCIZIO

$$\frac{\sqrt[2x-12]{4^{3x}}}{} = \frac{4}{}$$

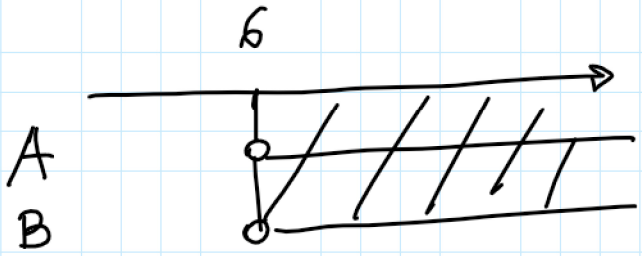
$\sqrt{2^{x-1}}$ $\qquad$ $\sqrt[x-6]{8^{6+x}}$

A $\begin{cases} 2x-12 > 0 \\ x-6 > 0 \end{cases}$ B

A $\quad 2x-12>0 \quad 2x>12 \quad x>6$

B $\quad x-6>0 \quad x>6$

GIF



DOMINIO $\quad x>6$

$$\frac{\sqrt[2x-12]{(2^2)^{3x}}}{2^{\frac{x-1}{2}}} = \frac{2^2}{\sqrt[x-6]{(2^3)^{x+6}}}$$

$$\frac{2^{\frac{6x}{2x-12}}}{2^{\frac{x-1}{2}}} = \frac{2^2}{2^{\frac{3x+18}{x-6}}}$$

$$2^{\frac{6x}{2x-12}-\frac{x-1}{2}} = 2^{2-\frac{3x+18}{x-6}}$$

$x_1 \quad x_2$

$a^{x_1} = a^{x_2} \Rightarrow x_1 = x_2$

$$\frac{6x}{2x-12} - \frac{x-1}{2} = 2 - \frac{3x+18}{x-6}$$

$$\frac{3\cancel{6}x}{\cancel{2}(x-6)} - \frac{x-1}{2} = \frac{2}{1} - \frac{3x+18}{(x-6)}$$

$2(x-6)$ $\quad 2 \quad 1 \quad (x-6)$

M.C.m.

$$\frac{6x-(x-1)(x-6)}{2(x-6)} = \frac{4(x-6)-2(3x+18)}{2(x-6)}$$

D: $x-6 \neq 0 \quad x \neq 6$

$$6x-(x^2-6x-x+6) = 4x-24-6x-36$$

$$6x-x^2+6x+x-6-4x+24+6x+36=0$$

$$-x^2+15x+54=0$$

SOMMA PRODOTTO

$$1x^2-15x-54=0$$

$-18 \oplus +3 = -15$

$-18 \odot +3 = -54$

AI 2 NUMERI CAMBIO IL SEGNO

$$(x-18)\cdot(x+3)=0$$

PRODOTTO E ZERO

SOL $x=18$ SI
$x=-3$ NO

"USO LAP"

$x-18=0 \rightarrow x=18$ SI

$x+3=0 \rightarrow x=-3$ NO

PER VOI

1) $3^x \cdot 9 = \frac{1}{3}$ SOL $x = -3$ 2) $\sqrt{2\sqrt{2}} = 4^{1-x}$ SOL $x = \frac{5}{8}$

3) $\frac{3^{x-1}}{9} = \frac{27^{1-x}}{3^{2+x}}$ SOL $x = \frac{4}{5}$ 4) $\sqrt[x-1]{125} \cdot \sqrt[x+2]{5^8} = \sqrt[x-1]{25} \cdot \sqrt[2x-1]{5^9}$

SOL $x = 2$

5) $3^{x+1} - \frac{3^x}{9} + 3^x = 35$ SOL $x = 2$

6) $4^x + 2^{x+2} - 12 = 0$ SOL $x = 1$ 7) $3^{x^2+x} = 1$ SOL $x = 0$ $x = 1$

DISEQUAZIONI

FUNZIONANO CON LO STESSO PRINCIPIO DELLE EQUAZIONI CON 2 DIFFERENZE

1) $a^{x_1} > a^{x_2}$ $a > 1$ $x_1 > x_2$

$0 < a < 1$ $x_1 < x_2$

OCCHIO AD a, SCRIVI SEMPRE $a \geq 1$ o $0 < a < 1$ PRIMA DI PASSARE AGLI ESPONENTI

2) $a^x > k$ $a^x \geq k$ $a^x < k$ $a^x \leq k$

ATTENTI SE k NEGATIVO O SE k ZERO

⇒ RISULTATO → IMP / → $\forall x \in \mathbb{R}$

RICORDA CHE $a^x > 0$ $\forall x \in \mathbb{R}$

$5^x > -3$ $\forall x \in \mathbb{R}$ $7^x \geq -9$ $\forall x \in \mathbb{R}$ → $\forall x \in \mathbb{R}$

$5^x > -3 \quad \forall x \in \mathbb{R}$ $\qquad$ $7^x \geq -9 \quad \forall x \in \mathbb{R}$ $\qquad$ $> \geq \quad \forall x \in \mathbb{R}$

$5^{-x} > 0 \quad \forall x \in \mathbb{R}$ $\qquad$ $7^x \geq 0 \quad \forall x \in \mathbb{R}$

$5^x < -3$ IMP $\qquad$ $7^x \leq -9$ IMP $\qquad$ $< \leq$ IMP

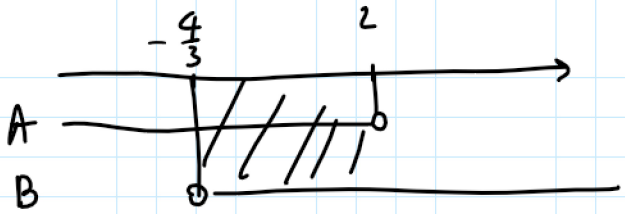
$5^{-x} < 0$ IMP $\qquad$ $7^x \leq 0$ IM

PUÒ ACCADERE CHE CON IL "DEPONGO" OTTENGO UN COMPRESO: IL COMPRESO GENERA SEMPRE UN SISTEMA → "GIF"

ovvero $\quad -6 < 3x - 2 < 4$

$$\begin{cases} A: & 3x - 2 < 4 \\ B: & 3x - 2 > -6 \end{cases} \qquad \begin{array}{lll} A & 3x < 6 & x < 2 \\ B & 3x > -4 & x > -\frac{4}{3} \end{array}$$

"GIF"



$$-\frac{4}{3} < x < 2$$

OPPURE

$$-6 < 3x - 2 < 4$$

$$-4 < 3x < 6$$

$-\frac{4}{3} < x < 2$

$1 < 2 - 4x < 5$

$-1 < -4x < 3$

CAMBIO SEGNO E VERSO

$1 > 4x > -3$ "RIGIRO"

$-3 < 4x < 1$

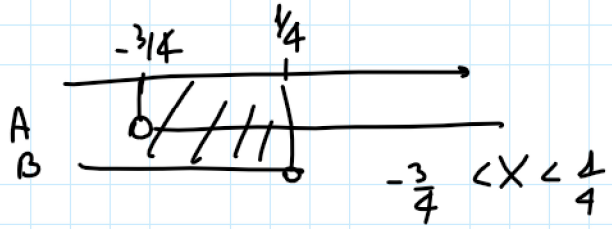
$-\frac{3}{4} < x < \frac{1}{4}$

OPPURE

$1 < 2 - 4x < 5$

A $2-4x<5$ A $-4x<3$ $4x>-3$ $x>-\frac{3}{4}$

B $2-4x>1$ B $-4x>-1$ $4x<1$ $x<\frac{1}{4}$



$-\frac{3}{4} < x < \frac{1}{4}$

ESERCIZI

$\left(\frac{2}{5}\right)^{x+3} < \left(\frac{5}{2}\right)^{x-2}$

INVERTO LA 2° BASE E CAMBIO SEGNO A TUTTO L'ESPONENTE

$x+3$ x

TUTTO L'ESPONENTE

$$\left(\frac{2}{5}\right)^{X+3} < \left(\frac{2}{5}\right)^{-X+2}$$

OBBLIGO DI SCRIVERE

$0 < a < 1$

$$X+3 > -X+2$$

$$2X > -1 \quad X > -\frac{1}{2}$$

$\emptyset$

$$\frac{4}{2^x-4} - \frac{2}{2^x-2} \leq 0$$

$2^x = t$

$$\frac{4}{(t-4)} - \frac{2}{(t-2)} \leq 0$$

$$\frac{4(t-2)-2(t-4)}{(t-4)(t-2)} \leq 0 \qquad \frac{4t-8-2t+8}{(t-4)(t-2)} \leq 0$$

$$\frac{2t}{(t-4)(t-2)} \leq 0$$

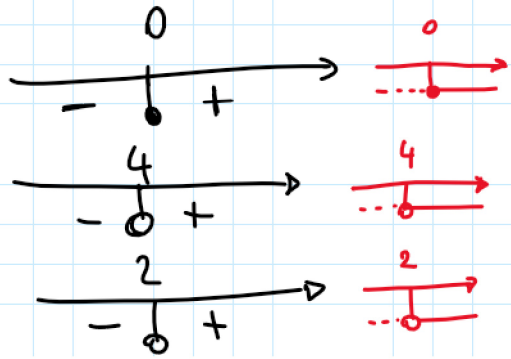
"S.S."

STUDIO SEGNO

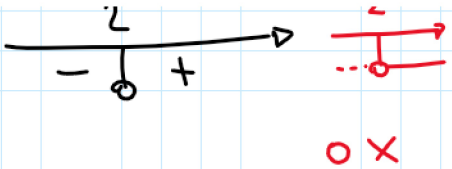
$N \geq 0 \quad 2t \geq 0 \quad t \geq 0$

$D_1 > 0 \quad t-4 > 0 \quad t > 4$

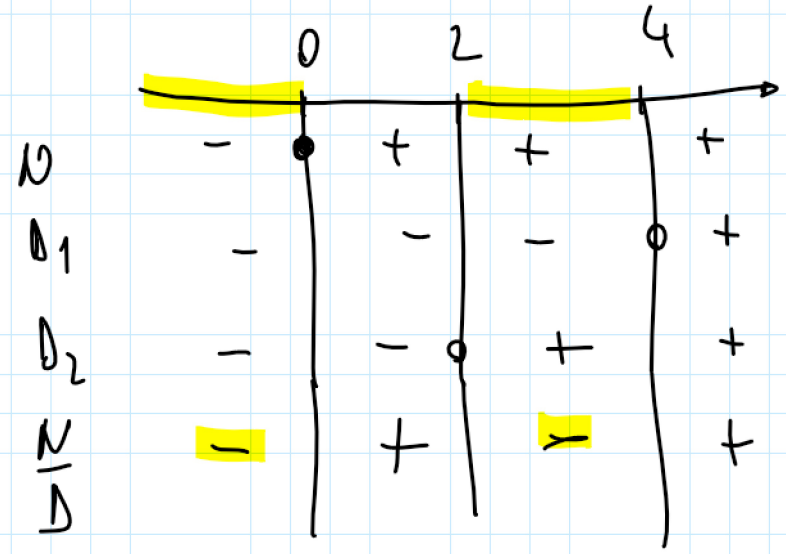
$D_2 > 0 \quad t-2 > 0 \quad t > 2$



$D_2 > 0 \quad t-2>0 \quad t>2$



o x



$t \leq 0 \quad \vee \quad 2 < t < 4$

$+$

$2^x \leq 0 \quad \vee \quad 2 < 2^x < 4$

OCCHIO IMP

$2^1 < 2^x < 2^2$ N.B. $a>1$

$1 < x < 2$

PER VOI

1) $3^{2x+2} < \frac{1}{3}$ Sol $x < -\frac{3}{2}$

2) $\left(\frac{2}{3}\right)^{x^2-2x} < \frac{3}{2}$ sol $\mathbb{R} \setminus \{1\}$

3) $\frac{2^x \cdot 8}{4^x} > \frac{16^{-x}}{8}$ sol $x > -2$

4) $9^x - 12 \cdot 3^x + 27 < 0$ sol $1 < x < 2$

5) $\frac{\left(\frac{1}{2}\right)^x - 4}{2^x} < 0$ sol $-2 < x < 2$

$$\frac{(2)^{-1}}{9-3^x} < 0 \qquad \text{sol } -2<x<2$$

DEFINIZIONE DI LOGARITMO

IL LOGARITMO IN BASE a DI b È L'ESPONENTE DA DARE AD a PER OTTENERE b

$\log_a b = c \iff a^c = b$ OPPURE $b = a^c$

b → ARGOMENTO; c → RISULTATO; a → BASE

$\log_2 4 = c \Rightarrow 2^c = 4 \Rightarrow 2^c = 2^2 \Rightarrow c = 2$

$\log_3 \frac{1}{3} = c \qquad 3^c = \frac{1}{3} \Rightarrow 3^c = 3^{-1} \Rightarrow c = -1$

ATTENTI

$\log_a 1 = c \qquad a^c = 1 \qquad a^c = a^0 \qquad c = 0$ SE L'ARGOMENTO È 1 IL LOG VALE 0

$\log_a a = c \qquad a^c = a \qquad c = 1$ SE L'ARGOMENTO = BASE IL LOG VALE 1

$\log_a b = c \Rightarrow b = \boxed{a^c}$ ESPONENZIALE $\quad a>0,\ a\neq 1$

a^c POSITIVO SEMPRE

IL LOGARITMO HA UN DOMINIO

ARGOMENTO > 0

$b > 0$

SCRITTURE SPECIALI

$\log_e = \ln$ (logaritmo naturale)

$\log_{10} = \log$ (" decimale)