

INTEGRALI GENERALIZZATI

AL FINITO

$$\int_a^b f(x) dx$$

a e/o $b \notin D$

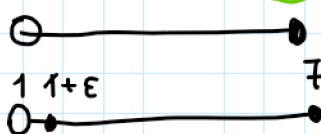
es $\int_{\boxed{1}}^{\boxed{7}} \frac{1}{x-1} dx$

$$f(x) = \frac{1}{x-1}$$

$$D: x \neq 1$$

$$\boxed{1} \notin D$$

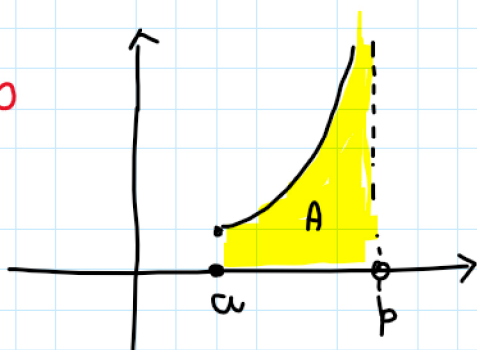
$$\boxed{7} \in D$$



$$\int_{1+\epsilon}^7 \frac{1}{x-1} dx = F(7) - F(1+\epsilon)$$

$$\lim_{\epsilon \rightarrow 0^+} (F(7) - F(1+\epsilon)) \begin{cases} \text{FINITO:} \\ \text{L'INTEGRALE CONVERGE} \\ \infty \text{ DIVERGE} \end{cases}$$

FINITO



ALL'INFINITO

ALMENO UNO DEI
2 ESTREMI È ∞

es $\int_5^{+\infty} \frac{1}{x-1} dx$

$$\int_5^M \frac{1}{x-1} dx = F(M) - F(5)$$

$$\lim_{M \rightarrow +\infty} [F(M) - F(5)]$$

$= l$ CONVERGE
 $= \infty$ DIVERGE

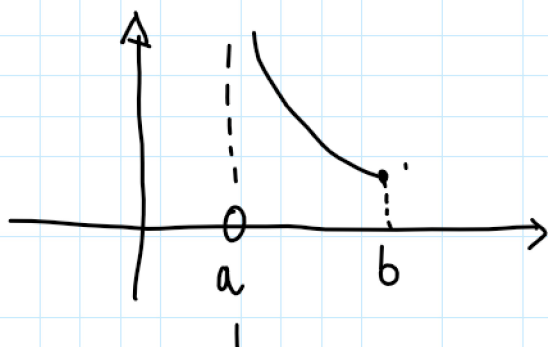
A? FINITA / INFINITA

CRITERIO $\int_a^b \frac{1}{(b-x)^\alpha} dx$

CONVERGE $\alpha < 1$

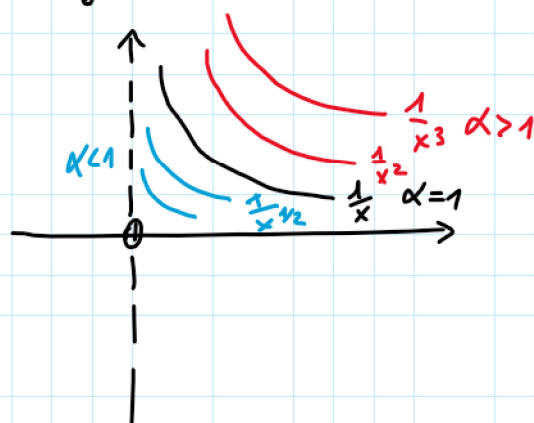
DIVERGE $\alpha \geq 1$

$\left(\sum \frac{1}{n^\alpha} \begin{cases} \text{CONVERGE } \alpha > 1 \\ \text{DIVERGE } \alpha \leq 1 \end{cases} \right)$

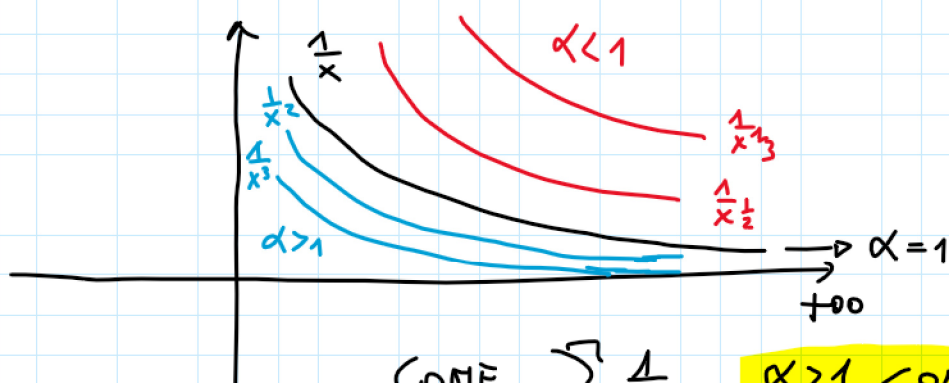


$\int_a^b \frac{1}{(x-a)^\alpha} dx$ $\begin{cases} \text{CONVERGE } \alpha < 1 \\ \text{DIVERGE } \alpha \geq 1 \end{cases}$

$f(x) = \frac{1}{x}$ $D: x \neq 0$



INFINITO



COME $\sum \frac{1}{n^\alpha}$

$\alpha > 1$ CONV
 $\alpha \leq 1$ DIV

CRITERIO CONFRONTO ASINTOTICO

$\int_a^b f(x) dx$ se $f(x) \sim g(x)$ nell'intorno di a o di b

$$\int_a^b f(x) dx \sim \int_a^b g(x) dx \quad \begin{array}{l} \text{(diverge)} \\ \text{se converge} \end{array} \int_a^b g(x) \quad \begin{array}{l} \text{(diverge)} \\ \text{converge} \end{array}$$

anche $\int_a^b f(x) dx$

$$\int_a^{+\infty} f(x) dx \quad \text{se } f(x) \sim g(x) \quad \text{come sopra}$$

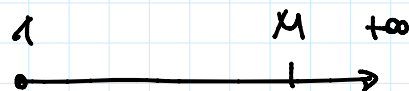
Esercizio

DATO $\int_1^{+\infty} \frac{1}{\sqrt{(x+1)^3}} dx$ STABILIRE SE CONVERGE O DIVERGE. NEL CASO IN CUI CONVERGE CALCOLARLO

ALL'INFINITO (USO $\sum \frac{1}{n^\alpha}$) $\alpha > 1$ CONV
 $\alpha \leq 1$ DIV

$$x \rightarrow +\infty \quad x+1 \sim x \quad \frac{1}{\sqrt{(x+1)^3}} \sim \frac{1}{\sqrt{x^3}} = \frac{1}{x^{3/2}} \quad \alpha > 1 \text{ CONV.}$$

SI COME CONVERGE VA CALCOLATO



$$\int_1^M \frac{1}{\sqrt{(x+1)^3}} dx$$

CALCOLO $\int \frac{1}{\sqrt{(x+1)^3}} dx = \int \frac{1}{(x+1)^{3/2}} dx$

$$= \int 1 (x+1)^{-3/2} dx$$

1° modo

$$\int f'(x) (f(x))^m dx$$

2° modo

$$\boxed{x+1 = t}$$

$$f(x) = x+1$$

$$f'(x) = 1$$

$$n = -\frac{3}{2}$$

$$\frac{(x+1)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + c$$

derivate

$$1 dx = 1 dt$$

$$\int t^{-\frac{3}{2}} dt$$

$$\frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + c = \frac{(x+1)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + c$$

$$\int \frac{1}{\sqrt{(x+1)^3}} dx = \frac{(x+1)^{-\frac{1}{2}}}{-\frac{1}{2}} + c = \frac{-2}{(x+1)^{\frac{1}{2}}} + c = \frac{-2}{\sqrt{(x+1)}} + c$$

Lo PORTO GU
INVERTO

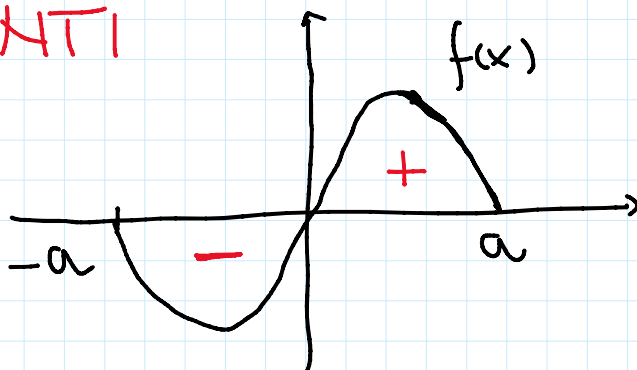
$$\int_1^M \frac{1}{\sqrt{(x+1)^3}} dx = \left[-\frac{2}{\sqrt{x+1}} \right]_1^M = F(M) - F(1)$$

$$= -\frac{2}{\sqrt{M+1}} - \left(-\frac{2}{\sqrt{2}} \right)$$

$$F(M) - F(1) = \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{M+1}}$$

$$\lim_{M \rightarrow +\infty} \left(\frac{2}{\sqrt{2}} - \frac{2}{\sqrt{M+1}} \right) = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$$

ATTENTI



$f(x)$ DISPARI

$$f(-x) = -f(x)$$

$$f(x) \text{ e } f(-x)$$

SONO OPPOSTI

$$\int_{-a}^a f(x) = 0$$

ESERCIZIO

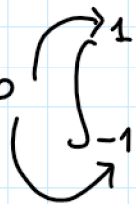
DATO $\int_0^1 \frac{1}{\sqrt[3]{\sin x}} dx$

VERIFICARE SE CONVERGE,
IN CASO AFFERMATIVO CALCOLARLO

DATO $\int_{-1}^1 \frac{1}{\sqrt[3]{\sin x}} dx$

VERIFICARE SE CONVERGE,
IN CASO AFFERMATIVO CALCOLARLO

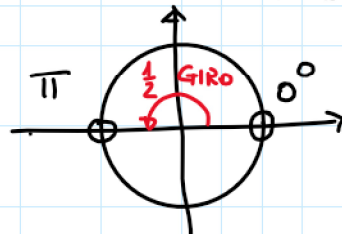
SIAMO AL FINITO



ALLORA CALCOLO IL DOMINIO

LA $\sqrt[3]{}$ DISPARI: $\forall x \in \mathbb{R}$

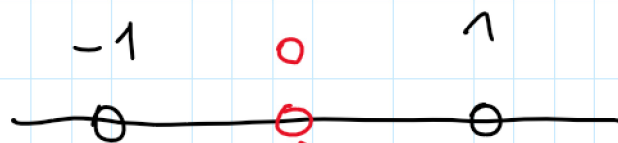
DENOMINATORE $\sin x \neq 0$



$$x \neq 0 + \frac{1}{2} \text{ GIRO}$$

$$x \neq 0 + k\pi$$

$$x \neq k\pi$$



IL PROBLEMA SOLO IN 0

ASINTOTICITA'

$$x \rightarrow 0 \quad \sin x \sim x$$

$$\frac{1}{\sqrt[3]{\sin x}} \sim \frac{1}{\sqrt[3]{x}} = \frac{1}{x^{\frac{1}{3}}}$$

$$\alpha = \frac{1}{3} < 1 \quad \text{CONV.}$$

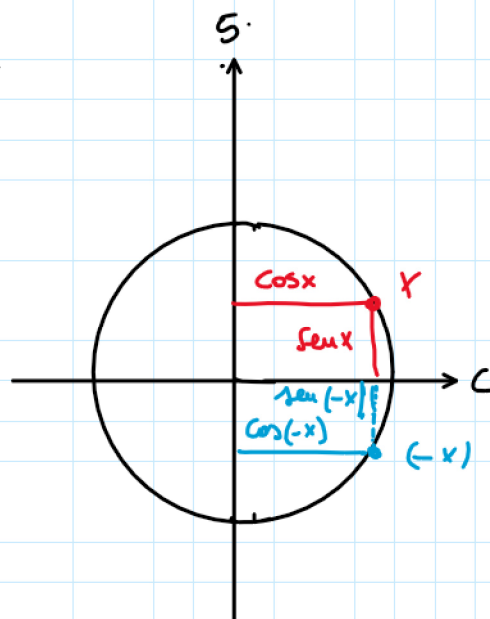
VA CALCOLATO

$$\int_{-1}^1 \frac{1}{\sqrt[3]{\sin x}} dx$$

PROVO A VEDERE SE $f(x)$ È DISPARI

$$\cos x = \cos(-x) \quad \cos x \text{ PARI}$$

$$\sin x = -\sin(-x) \quad \sin x \text{ DISPARI}$$



$$f(x) = \frac{1}{\sqrt[3]{\sin x}}$$

$$f(-x) = \frac{1}{\sqrt[3]{\sin(-x)}} = \frac{1}{\sqrt[3]{-\sin x}}$$

$$= \frac{1}{-\sqrt[3]{\sin x}} = -\frac{1}{\sqrt[3]{\sin x}}$$

$f(x)$ è DISPARI

$$\Rightarrow \int_{-1}^1 \frac{1}{\sqrt[3]{\sin x}} dx = 0$$

$$\int_{-\sqrt{3}}^0 \frac{|x+1|}{x^2+1} dx$$

$D: x^2+1 \neq 0 \quad \forall x \in \mathbb{R}$
NO GENERALIZZATO

ROMPE $|x+1|$

$$x+1 \geq 0 \quad x \geq -1$$

$$-\sqrt{3} \quad -(x+1) \quad -1 \quad (x+1) \quad 0$$

$$\begin{array}{c} -1 \\ \hline - \quad | \quad + \\ \hline -(x+1) \quad | \quad (x+1) \end{array}$$

$$\int_{-\sqrt{3}}^{-1} -\frac{(x+1)}{x^2+1} dx + \int_{-1}^0 \frac{x+1}{x^2+1} dx$$

CALCOLO $\int \frac{x+1}{x^2+1} dx = \int \frac{x}{x^2+1} dx + \int \frac{1}{x^2+1} dx$

$$\frac{1}{2} \int \frac{2x}{x^2+1} dx + \arctan x$$

$$f(x) = x^2+1$$

$$f'(x) = 2x$$

$$\frac{1}{2} \ln(x^2+1) + \arctan x + c$$

-1

0

$$- \int_{-\sqrt{3}}^{-1} \frac{x+1}{x^2+1} dx + \int_{-1}^0 \frac{x+1}{x^2+1} dx$$

$$- \left[\frac{1}{2} \ln(x^2+1) + \arctan x \right]_{-\sqrt{3}}^{-1} + \left[\frac{1}{2} \ln(x^2+1) + \arctan x \right]_{-1}^0$$

$$- \left(\left(\frac{1}{2} \ln 2 + \arctan(-1) \right) - \left(\frac{1}{2} \ln 4 + \arctan(-\sqrt{3}) \right) \right) + \left(\left(\frac{1}{2} \ln 1 + \arctan 0 \right) - \left(\frac{1}{2} \ln 2 + \arctan(-1) \right) \right)$$

