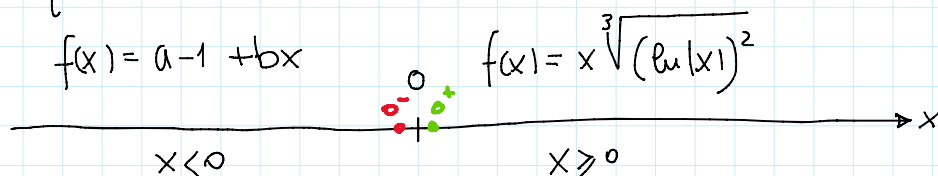


APPELLO 12.01. 2017

DIRE PER QUALI VALORI DI a e b $f(x)$ È CONTINUA E DERIVABILE $\forall x$

$$f(x) = \begin{cases} a-1+bx & \text{se } x < 0 \\ x \sqrt[3]{(\ln|x|)^2} & \text{se } x \geq 0 \end{cases}$$

FUNZIONE "A TRATTI" PER CASI



$$D: \forall x < 0$$

1° OSSERVAZIONE STO IN $x \geq 0 \Rightarrow |x| = x$

$$f(x) = x \sqrt[3]{(\ln x)^2}$$

D $f(x)$ $\sqrt[3]{\quad}$ NON MI DA PROBLEMI
 $\ln x$ MI DA PROBLEMI

DEVO CAPIRE COSA ACCADE IN $x=0$

LA FUNZIONE $f(x)$ SARA' CONTINUA SE $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$ FINITI

$$\lim_{x \rightarrow 0^-} (a-1+bx) = \boxed{a-1} \quad \forall b$$

$$\lim_{x \rightarrow 0^+} x \sqrt[3]{(\ln x)^2} \text{ (ABBIAMO DIMOSTRATO) } = \boxed{0} \quad (x = e^{-t})$$

$$\boxed{a-1} = \boxed{0}$$

$a=1$. LA FUNZIONE $f(x)$ È CONTINUA SE $a=1 \quad \forall b$

$$f(x) = \begin{cases} bx & x < 0 \\ x \sqrt[3]{(\ln x)^2} & x \geq 0 \end{cases} \quad x < 0 \cup x \geq 0 \quad \forall x \in \mathbb{R} \quad ? \quad \boxed{x=0}$$

$$f(x) = bx \quad 0 \quad f(x) = x \cdot (\ln x)^{\frac{2}{3}}$$

DEVO DERIVARE

$$f'(x) = b \cdot 1 = b \quad \left\{ \begin{array}{l} \rightarrow D \left[(f(x))^m \right] = m \left(\underbrace{f(x)} \right)^{m-1} \cdot \underbrace{f'(x)} \end{array} \right.$$

$$f'(x) = b \cdot 1 = b$$

$$\rightarrow \text{TEO PRODOTTO} = (f(x))' \cdot g(x)$$

$$f'(x) = 1 \cdot (\ln x)^{\frac{2}{3}} + \cancel{x} \cdot \frac{2}{3} (\ln x)^{\frac{2}{3}-1} \cdot \frac{1}{x}$$

$$f'(x) = (\ln x)^{\frac{2}{3}} + \frac{2}{3} (\ln x)^{-\frac{1}{3}} \rightarrow \text{DERIVATO } \ln x \rightarrow a^{-m} = \frac{1}{a^m}$$

$$f'(x) = \sqrt[3]{(\ln x)^2} + \frac{2}{3} \cdot \frac{1}{(\ln x)^{\frac{1}{3}}}$$

$$f'(x) = \sqrt[3]{(\ln x)^2} + \frac{2}{3 \sqrt[3]{\ln x}}$$

$$f'(x) = \frac{\sqrt[3]{(\ln x)^3} + 2}{3 \sqrt[3]{\ln x}}$$

$$f'(x) = b \quad \text{se } x < 0$$

$$f'(x) = \frac{3 \ln x + 2}{3 \sqrt[3]{\ln x}} \quad \text{se } x > 0$$

$$\text{DEN } \ln x \neq 0 \quad x \neq 1$$

$$\lim_{x \rightarrow 0^-} f'(x) = b$$

$$\lim_{x \rightarrow 0^+} \frac{3 \ln x + 2}{3 \sqrt[3]{\ln x}} = \frac{\infty}{\infty} \quad \text{F.O.I.}$$

$$(\ln 0^+ = -\infty)$$

$$\text{PONGO: } \ln x = t \quad \text{se } x \rightarrow 0^+ \quad \ln x \rightarrow -\infty$$

$$\lim_{t \rightarrow -\infty} \frac{3t + 2}{3t} = \frac{\infty}{\infty}$$

LA FUNZIONE NON È DERIVABILE
PER NESSUN VALORE DI b

$$\frac{(-\infty)^1}{\sqrt[3]{-\infty}} = \frac{-\infty}{-\infty} = +$$

$$\lim_{x \rightarrow 0^-} f'(x) = b$$

FACCIAMO FINTA CHE

$$\lim_{x \rightarrow 0^+} f'(x) = \frac{1}{2}$$

$$b = \frac{1}{2} \quad \text{DERIVABILE}$$

APPELLO DEL 04.02.2020

AL VARIARE DI α E $\beta \in \mathbb{R}$ STUDIA CONTINUITÀ E DERIV.

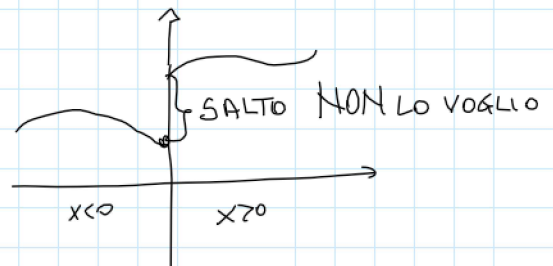
$$f(x) = \begin{cases} \frac{\ln(1+4x+2x^2) - 4x}{x} & \text{se } x > 0 \\ \alpha & \text{se } x = 0 \\ \frac{\beta(e^{x^2} - 1)}{x} & \text{se } x < 0 \end{cases}$$

$f(0) = \alpha$

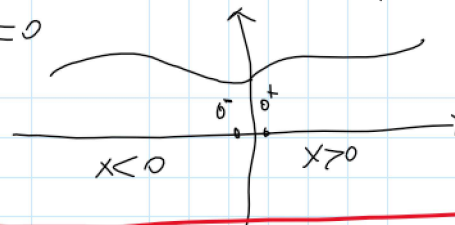
$f(x) = \frac{\ln(1+4x+2x^2) - 4x}{x}$

$x < 0 \quad 0 \quad x > 0$

NON VOGLIO CHE ACCADA IN $x=0$



VOGLIO CHE IN $x=0$

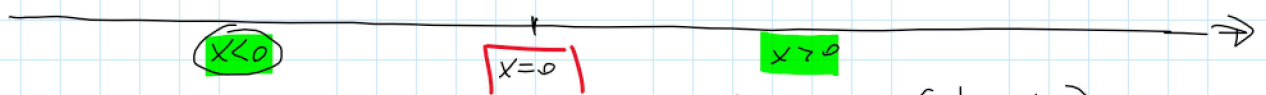


$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$f(x) = \frac{\beta(e^{x^2} - 1)}{x}$$

$$f(0) = \alpha$$

$$f(x) = \frac{\ln(1+4x+2x^2) - 4x}{x}$$



D $x \neq 0$
SICCOME "LA USO" PER
 $x < 0$ STO A POSTO

$$D \begin{cases} x \neq 0 & (\text{denom.}) \\ 1+4x+2x^2 > 0 & (\text{argom. ln}) \end{cases}$$

$$2x^2 + 4x + 1 > 0$$

$$2x^2 + 4x + 1 = 0 \quad \Delta = 16 - 4(2) = 16 - 8 = 8$$

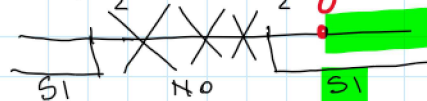
$$x_{1/2} = \frac{-4 \pm 2\sqrt{2}}{4}$$

$$= -1 \pm \frac{\sqrt{2}}{2}$$

$$-1 - \frac{\sqrt{2}}{2}$$

$$x_1 = -1 - \frac{\sqrt{2}}{2}$$

$$x_2 = -1 + \frac{\sqrt{2}}{2}$$



DEVO SOLO "TESTARE" $x=0$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

STO NELL'INTORNO DI 0 ($x \rightarrow 0$)

ASINTOTICITA'

$$e^{f(x)} - 1 \sim f(x) \quad \text{se } f(x) \rightarrow 0$$

$$\ln(1 + f(x)) \sim f(x) \quad \text{se } f(x) \rightarrow 0$$

$$e^{x^2} - 1 \sim x^2 \quad \text{perché } x^2 \rightarrow 0 \text{ se } x \rightarrow 0$$

$$\ln(1 + \boxed{4x + 2x^2}) \sim \boxed{4x + 2x^2} \quad \text{perché } 4x + 2x^2 \rightarrow 0 \text{ se } x \rightarrow 0$$

$$f(x) = \begin{cases} \frac{4x + 2x^2 - 4x}{x} & x > 0 \\ \alpha & x = 0 \rightarrow f(0) = \alpha \\ \beta(x^2) & x < 0 \end{cases}$$

$$f(x) = \begin{cases} 2x & x > 0 \\ \alpha & x = 0 \\ \beta x & x < 0 \end{cases}$$

$$\lim_{x \rightarrow 0^-} \beta x = 0 \quad \lim_{x \rightarrow 0^+} 2x = 0 \quad f(0) = \alpha$$

$$\boxed{\alpha = 0}$$

se $\alpha = 0$ $f(x) \Rightarrow$ CONTINUA

$$\begin{array}{c} \beta x \qquad \qquad 2x \\ \hline x < 0 \qquad \quad x > 0 \\ f'(x) = \beta \qquad f'(x) = 2 \end{array}$$

$$\lim_{x \rightarrow 0^-} f'(x) = \lim_{x \rightarrow 0^+} f'(x) \\ \beta = 2$$

se $\beta = 2$ $f(x) \Rightarrow$ DERIVABILE

APPELLO 04.02.2020

1-x

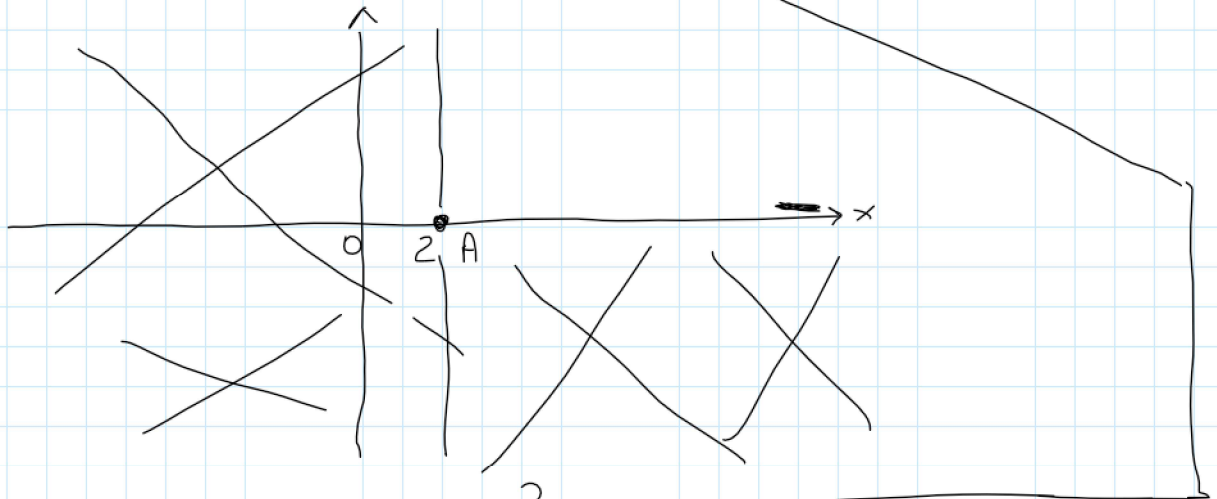
non si può derivare $f(x) = 0$ in $x = 0$ $\sqrt{x-1}$ TRAMIA //

APPELLO 04.02.2020

Dopo aver disegnato $f(x) = e^{1-x} \cdot \sqrt{x-2}$ TROVA IL
NUMERO DELLE SOLUZIONI DI $e^{1-x} \cdot \sqrt{x-2} = \alpha \quad \alpha \in \mathbb{R}$
AL VARIARE DI α

$$f(x) = e^{1-x} \cdot \sqrt{x-2}$$

1) DOMINIO $x-2 \geq 0 \quad x \geq 2$



2) INT. ASSI

$$\begin{cases} x=0? \\ \text{NO} \end{cases}$$

$$\begin{cases} y=0 \\ e^{1-x} \cdot \sqrt{x-2} = 0 \end{cases}$$

PER $\uparrow$ ZERO $\uparrow$ SEPARO

$$e^{1-x} = 0 \quad \text{IMP} \quad | \quad e^{\text{fu}} \oplus$$

$$\sqrt{x-2} = 0 \quad x=2$$

$A(2, 0)$

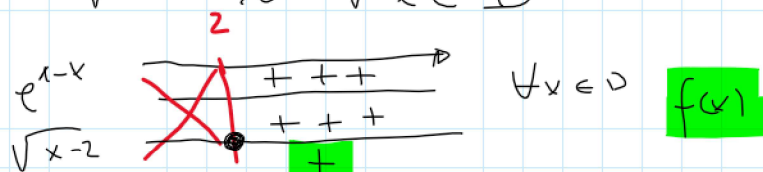
3) STUDIO SEGNO

$$e^{1-x} \cdot \sqrt{x-2} \geq 0$$

SCEGLI TU ≥ 0

$$e^{1-x} > 0 \quad \forall x \in \mathbb{D}$$

$$\sqrt{x-2} \geq 0 \quad \forall x \in \mathbb{D}$$



N.B. NELLO STUDIO DI FUNZIONE OGNI DISEGNO CHE
FATE (STUDIO SEGNO, STUDIO DI $f'(x)$, STUDIO $f''(x)$)
PREVEDE CHE "CANCELLATE"
f(x) f'(x) f''(x)

PREVEDE CHE "CANCELLATE"
LE PARTI "CANCELLATE" NEL DOMINIO,
ALTRIMENTI ACCETTIAMO INTERVALLI CHE NON FANNO PARTE
DEL DOMINIO (CHE INVECE COMANDA)

4) ASINTOTI

$x \rightarrow 2$ $\rightarrow \infty$

NO VERTICALI PERCHÉ $2 \in D$ E NON HO TOLTO $\begin{pmatrix} 0 \\ \infty \end{pmatrix}$
ORIZZ. NESSUN VALORE FINITO.

$$\lim_{x \rightarrow +\infty} e^{1-x} \cdot \sqrt{x-2}$$

$$\lim_{x \rightarrow +\infty} e^{1-x} \cdot \sqrt{x-2} = 0$$

$$\left(\begin{array}{c} 1-x \\ e^{-x} = \frac{1}{e^x} \end{array} \right)$$

$\rightarrow e^{x-1}$ "PIÙ POTENTE" $\sqrt{x-2}$

$y=0$ AS. ORIZZ.



5) max e min

$$f(x) = e^{1-x} \cdot \sqrt{x-2}$$

"RICORDA

$$D[e^{f(x)}] = e^{f(x)} \cdot f'(x)$$

$$D[\sqrt{f(x)}] = \frac{f'(x)}{2\sqrt{f(x)}}$$

$$D(e^{1-x}) = e^{1-x} \cdot (-1) = -e^{1-x}$$

$$D(\sqrt{x-2}) = \frac{1}{2\sqrt{x-2}}$$

$$f'(x) = -e^{1-x} \cdot \sqrt{x-2} + e^{1-x} \cdot \frac{1}{2\sqrt{x-2}}$$

("PRODOTTO)

$$= e^{1-x} \left(-\sqrt{x-2} + \frac{1}{2\sqrt{x-2}} \right)$$

$$= e^{1-x} \left(\frac{-2\sqrt{(x-2)^2} + 1}{2\sqrt{x-2}} \right)$$

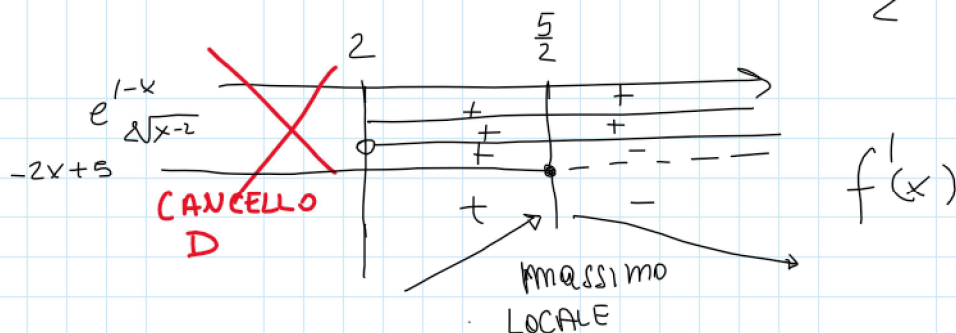
$$= e^{1-x} \left(\frac{-2x + 4 + 1}{2\sqrt{x-2}} \right) \geq 0$$

$$\begin{array}{l} 1-x \downarrow \\ e^{1-x} \geq 0 \\ \forall x \in D \end{array}$$

$$\begin{array}{l} \downarrow \\ 2\sqrt{x-2} > 0 \quad x > 2 \\ \text{DEN} \end{array}$$

$$-2x + 5 \geq 0$$

$$x \leq \frac{5}{2}$$



$$\begin{aligned} \left\{ \begin{array}{l} x = \frac{5}{2} \\ y = e^{1-\frac{5}{2}} \cdot \sqrt{\frac{5}{2}-2} = e^{-\frac{3}{2}} \cdot \sqrt{\frac{1}{2}} = e^{-\frac{3}{2}} \cdot \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ = e^{-\frac{3}{2}} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \cdot \frac{1}{e^{\frac{3}{2}}} = \frac{\sqrt{2}}{2\sqrt{e^3}} \end{array} \right. \end{aligned}$$

$$x=2 \in D$$

$$x=2 \notin D' \quad (D' \text{ dominio della derivata})$$

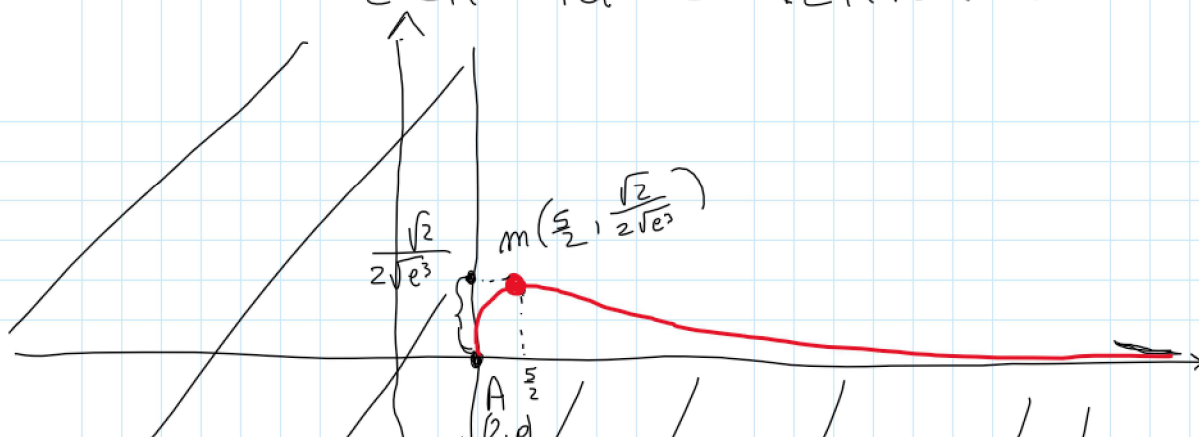
$$f'(x) = \frac{e^{1-x}(-2x+5)}{2\sqrt{x-2}}$$

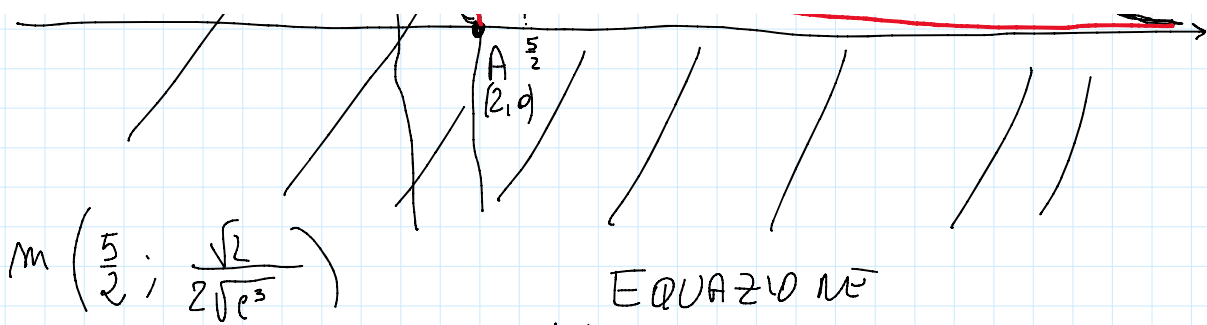
$$\oplus' \quad x-2 > 0 \quad x > 2$$

$$x = \begin{array}{l} \in D \\ \notin D' \end{array} \quad \text{STUDIO IL LIMITE } x \rightarrow 2^+ \text{ DI } f'(x)$$

$$\lim_{x \rightarrow 2^+} \frac{e^{1-x}(-2x+5)}{2\sqrt{x-2}} = \frac{e^{-1}(1)}{0^+} = (+\infty)$$

$x=2$ PUNTO NON DERIVABILITA'
E LA TG È VERTICALE



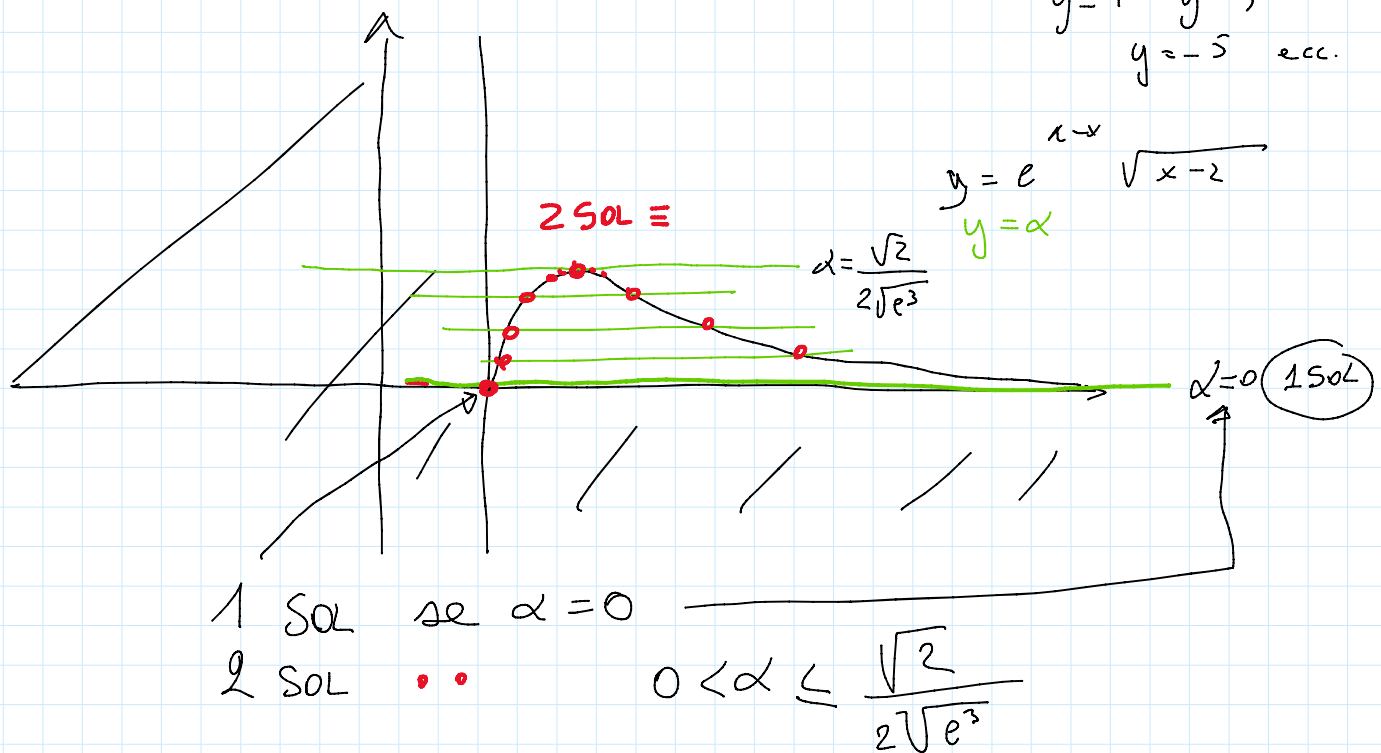
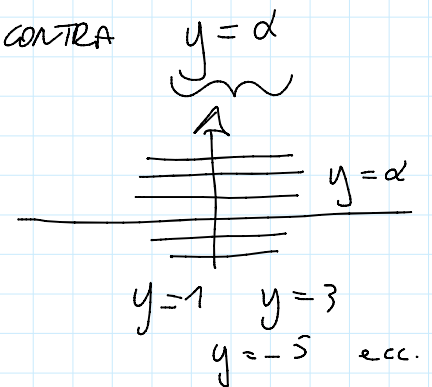


EQUAZIONE

$$\underbrace{e^{1-x} \cdot \sqrt{x-2}}_y = \underbrace{\alpha}_y$$

$$y = e^{1-x} \sqrt{x-2} \quad \text{IN CONTRA}$$

↓
DISEGNATA



APPELLO 07.01.2020

DETERMINARE IL PROLUNGAMENTO $\bar{f}(x)$ DI
 $f(x)$ SU TUTTO $\mathbb{R}$

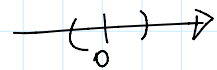
$$f(x) = \frac{3 \ln(1+x^2)}{\ln(e^{x^2}-1)}$$

$$\frac{1}{\sin(e^{x^2}-1)}$$

ln $\rightarrow 1+x^2 > 0 \quad \forall x$ (SOMMA DI 1 CON $\begin{pmatrix} x^2 \\ + \\ 0 \end{pmatrix}$)
 l'unico punto è $x=0$ CHE MI DA PROBLEMI

PERCHÉ SE $x=0$ $\sin(e^0-1) = \sin(0)=0$ (0 AL DEN)

STO NELL'INTORNO DI 0



ASINTOTICITA' $\ln(1+x^2) \sim x^2$
 $e^{x^2}-1 \sim x^2$

$$\lim_{x \rightarrow 0} \frac{3x^2}{\sin x^2} \quad \sin x^2 \sim x^2$$

$$\lim_{x \rightarrow 0} \frac{3x^2}{x^2} = 3 \quad I(0) \quad f(x) \text{ LIMITE } 3$$

PROVING.
$$\bar{f}(x) = \begin{cases} \frac{3 \ln(1+x^2)}{\sin(e^{x^2}-1)} & \text{se } x \neq 0 \\ 3 & \text{se } x=0 \end{cases}$$