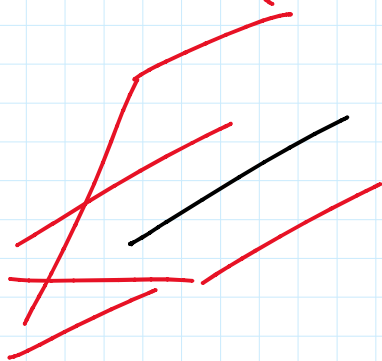


PARALLELISMO

TRA 2 RETTE $l = l' \quad m = m' \quad n = n'$ TRA 2 PIANI $a = a' \quad b = b' \quad c = c'$ TRA RETTA E PIANO $al + bm + cn = 0$

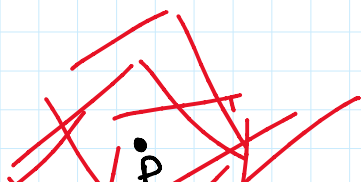
PERPENDICOLARITÀ

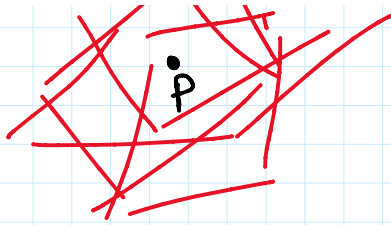
TRA 2 RETTE $ll' + mm' + nn' = 0$ TRA 2 PIANI $a \cdot a' + b \cdot b' + c \cdot c' = 0$ TRA RETTA E PIANO $a = l \quad b = m \quad c = n$ FASCIO DI PIANI (∞ PIANI CHE CONTENGONO UNA RETTA)

RETTA r :

$$\begin{cases} ax + by + cz + d = 0 \\ a'x + b'y + c'z + d' = 0 \end{cases}$$

FASCIO PIANI $\lambda(ax + by + cz + d) + \mu(a'x + b'y + c'z + d') = 0$

VANNO RICAVATI λ e μ STELLA DI PIANI (∞ PIANI PASSANTI PER UN PUNTO)



$P(x_0, y_0, z_0)$ STELLA $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

EQUAZIONE DEL PIANO

PER 3 PUNTI (NON ALLINEATI)

$A(x_A, y_A, z_A)$ $B(x_B, y_B, z_B)$ $C(x_C, y_C, z_C)$

$$\begin{vmatrix} x & y & z & 1 \\ x_A & y_A & z_A & 1 \\ x_B & y_B & z_B & 1 \\ x_C & y_C & z_C & 1 \end{vmatrix} = 0$$

POSIZIONE TRA 2 RETTE

$$r = \begin{cases} x = x_0 + lt \\ y = y_0 + mt \\ z = z_0 + nt \end{cases}$$

$$s = \begin{cases} x = x_1 + l't \\ y = y_1 + m't \\ z = z_1 + n't \end{cases}$$

$$\begin{vmatrix} x_0 - x_1 & y_0 - y_1 & z_0 - z_1 \\ l & m & n \\ l' & m' & n' \end{vmatrix} \begin{matrix} \nearrow \neq 0 \\ \searrow = 0 \end{matrix} \begin{matrix} \pi \text{ e } s \\ \text{SGHERME} \\ \pi \text{ e } s \\ \text{COPLANARI} \end{matrix}$$

$\swarrow$ INCIDENTI $\downarrow$ PARALLELE $\searrow$ COINCIDENTI

ANGOLI

TRA 2 RETTE r e r' $\cos \widehat{rr'} = \pm \frac{ll' + mm' + nn'}{\sqrt{l^2 + m^2 + n^2} \cdot \sqrt{l'^2 + m'^2 + n'^2}}$

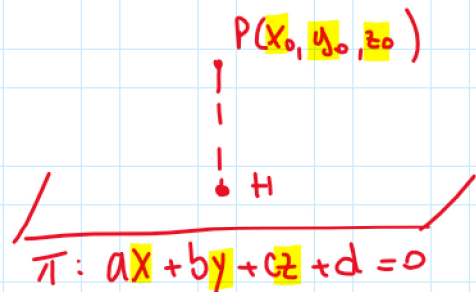
TRA 2 PIANI π e π' $\cos \widehat{\pi\pi'} = \pm \frac{aa' + bb' + cc'}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{a'^2 + b'^2 + c'^2}}$

TRA RETTA E PIANO r e π

$$\sin \widehat{r\pi} = \left| \frac{al + bm + cn}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{l^2 + m^2 + n^2}} \right|$$

DISTANZA PUNTO - PIANO

$$PH = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$



ESERCIZIO

TROVA L'EQUAZIONE DELLA RETTA ^{r} PASSANTE PER $O(0,0,0)$, PARALLELA AL PIANO $\alpha: x - 2y + z - 1 = 0$ E CHE FORMA UN ANGOLO DI $\frac{\pi}{6}$ CON IL PIANO XY .

PASSA PER $O(0,0,0)$ $r: \begin{cases} x = 0 + lt \\ y = 0 + mt \\ z = 0 + nt \end{cases}$ ①

r

π
 $\rightarrow$ PARALLELA AD α $\quad \begin{cases} z=0+mt \end{cases}$ $al + bm + cm = 0$ (2)
 $\rightarrow$ ANGOLO DI $\frac{\pi}{6}$ CON $z=0$

$$\textcircled{3} \quad \cos \frac{\pi}{6} = \left| \frac{al + bm + cm}{\sqrt{a^2+b^2+c^2} \cdot \sqrt{l^2+m^2+n^2}} \right|$$

$\textcircled{2}$
 Sono $\alpha: x-2y+z-1=0$
 $al + bm + cm = 0$
 Sono di π
 $\rightarrow a=1$ (NUMERO DI x)
 $\rightarrow b=-2$ (" DI y)
 $\rightarrow c=1$ (" DI z)

$$\textcircled{2} \quad \boxed{l - 2m + n = 0}$$

$\textcircled{3}$ $\frac{1}{2} = \left| \frac{al + bm + cm}{\sqrt{a^2+b^2+c^2} \cdot \sqrt{l^2+m^2+n^2}} \right|$ a, b, c DAL PIANO xy :
 $\text{PIANO } z=0 \begin{cases} a=0 \\ b=0 \\ c=1 \end{cases}$
 l, m, n sono di π

$$\frac{1}{2} = \left| \frac{0 \cdot l + 0 \cdot m + 1 \cdot n}{\sqrt{0^2+0^2+1^2} \cdot \sqrt{l^2+m^2+n^2}} \right|$$

$$\frac{1}{2} = \left| \frac{n}{\sqrt{l^2+m^2+n^2}} \right| \quad ()^2 = ()^2$$

$$\frac{1}{4} = \frac{n^2}{l^2+m^2+n^2}$$

$$\frac{l^2+m^2+n^2}{\dots} = \frac{4n^2}{\dots}$$

$$\frac{l + m^2 + m^2}{4(l^2 + m^2 + m^2)} = \frac{4m}{4(l^2 + m^2 + m^2)}$$

$$\textcircled{3} \quad \boxed{l^2 + m^2 - 3m^2 = 0}$$

$$\textcircled{2} \quad \begin{cases} \textcircled{l} - 2m + m = 0 \\ l^2 + m^2 - 3m^2 = 0 \end{cases} \quad \begin{cases} \textcircled{l} = 2m - m \\ (2m - m)^2 + m^2 - 3m^2 = 0 \end{cases}$$

$$\textcircled{3} \quad \begin{cases} l = 2m - m \\ 4m^2 - 4mm + m^2 + m^2 - 3m^2 = 0 \end{cases} \quad \begin{matrix} 5m^2 - 4mm - 2m^2 = 0 \\ \text{"TRUCCO"} \quad (\div m^2) \end{matrix}$$

$$5 \frac{m^2}{m^2} - \frac{4mm}{m^2} - 2 = 0$$

$$5 \left(\frac{m}{m} \right)^2 - 4 \left(\frac{m}{m} \right) - 2 = 0$$

$$\Delta = 16 - 4(5)(-2) = 16 + 40 = 56$$

$$\frac{m}{m} = \frac{4 \pm \sqrt{56}}{10} = \frac{4 \pm \sqrt{4 \cdot 14}}{10}$$

$$\frac{m}{m} = \frac{4 \pm 2\sqrt{14}}{10} = \frac{4}{10} \pm \frac{2\sqrt{14}}{10}$$

$$\frac{m}{m} = \frac{2}{5} \pm \frac{\sqrt{14}}{5}$$

PONGO PER COMODITÀ
m=1

$$\begin{cases} m=1 \\ m = \frac{2}{5} + \frac{\sqrt{14}}{5} \\ l = 2m - n \end{cases}$$

$$\begin{cases} m=1 \\ m = \frac{2}{5} - \frac{\sqrt{14}}{5} \\ l = 2m - n \end{cases}$$

$$\begin{cases} m=1 \\ m = \frac{2}{5} + \frac{\sqrt{14}}{5} \\ l = \frac{4}{5} + \frac{2\sqrt{14}}{5} - 1 \end{cases}$$

$$\begin{cases} m=1 \\ m = \frac{2}{5} - \frac{2\sqrt{14}}{5} \\ l = \frac{4}{5} - \frac{2\sqrt{14}}{5} - 1 \end{cases}$$

TROVA π PIANI CHE CONTENGONO

$$\pi: \begin{cases} 2x - 3z + 2 = 0 \\ y = 0 \end{cases} \quad \text{E FORNANO UN}$$

ANGOLO DI $\frac{\pi}{6}$ CON IL PIANO $\alpha: x - z = 0$

π CONTENGONO π : FASCIO $\textcircled{1}$
 $\lambda(2x - 3z + 2) + \mu(y) = 0$

ANGOLO $\frac{\pi}{6}$ CON α

$$\cos \widehat{\pi \alpha} = \pm \frac{aa' + bb' + cc'}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{a'^2 + b'^2 + c'^2}} \quad \textcircled{2}$$

$$\textcircled{1} \quad \underbrace{2\lambda}_a x + \underbrace{\mu}_b y + \underbrace{-3\lambda}_c z + \underbrace{2\lambda}_d = 0$$

$$(2) \quad \cos \frac{\pi}{6} = \pm \frac{aa' + bb' + cc'}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{a'^2 + b'^2 + c'^2}}$$

$$a = \underline{2\lambda} \quad b = \underline{\mu} \quad c = \underline{-3\lambda}$$

$$\alpha: x - z = 0 \quad a' = \underline{1} \quad b' = \underline{0} \quad c' = \underline{-1}$$

$$\frac{\sqrt{3}}{2} = \pm \frac{\underline{2\lambda} + \underline{0} + \underline{+3\lambda}}{\sqrt{4\lambda^2 + \mu^2 + 9\lambda^2} \cdot \sqrt{1 + 0 + 1}}$$

$$\frac{\sqrt{3}}{2} = \pm \frac{5\lambda}{\sqrt{13\lambda^2 + \mu^2} \cdot \sqrt{2}} \quad \left(\quad \right)^2 = \left(\quad \right)^2$$

$$\frac{3}{4} = \frac{25\lambda^2}{(13\lambda^2 + \mu^2) \cdot 2}$$

$$\frac{3(13\lambda^2 + \mu^2)}{4(13\lambda^2 + \mu^2)} = \frac{50\lambda^2}{4(13\lambda^2 + \mu^2)}$$

$$39\lambda^2 + 3\mu^2 = 50\lambda^2$$

$$3\mu^2 - 11\lambda^2 = 0$$

$$3\mu^2 = 11\lambda^2$$

$$\mu^2 = \frac{11}{3}\lambda^2$$

$$\mu = (\pm) \sqrt{\frac{11}{3}} \lambda$$

$$\mu = \pm \sqrt{\frac{11}{3}} \lambda$$

POSSO DARE UN VALORE A λ . (NO ZERO)

$$\lambda = \sqrt{3} \Rightarrow \mu = \pm \sqrt{\frac{11}{3}} \cdot \sqrt{3}$$

$$\mu = \pm \frac{\sqrt{11}}{\sqrt{3}} \cdot \sqrt{3} \quad \mu = \pm \sqrt{11}$$

$$\left. \begin{array}{l} \lambda = \sqrt{3} \quad \mu = \sqrt{11} \\ \lambda = \sqrt{3} \quad \mu = -\sqrt{11} \end{array} \right| 2\lambda x + \mu y - 3\lambda z + 2\lambda = 0$$

$$\pi_1 = 2\sqrt{3}x + \sqrt{11}y - 3\sqrt{3}z + 2\sqrt{3} = 0 \quad \left(\lambda = \sqrt{3} \quad \mu = \sqrt{11} \right)$$

$$\pi_2 = 2\sqrt{3}x - \sqrt{11}y - 3\sqrt{3}z + 2\sqrt{3} = 0 \quad \left(\lambda = \sqrt{3} \quad \mu = -\sqrt{11} \right)$$

TROVARE π PIANI $\perp$ ALLA RETTA

$$r: X + Z = Y - 2Z = 0 \quad \text{E AVENTI}$$

DISTANZA 2 DALL'ORIGINE

π $\rightarrow$ PERPENDICOLARI AD r $\begin{matrix} a=l \\ b=m \\ c=n \end{matrix}$ ①

Δ DISTANZA $(0,0,0)$ ORIGINE / PIANO = 2

$$\frac{|a \cdot 0 + b \cdot 0 + c \cdot 0 + d|}{\sqrt{a^2 + b^2 + c^2}} = 2 \quad \text{②}$$

① $\pi: ax + by + cz + d = 0$ $\begin{matrix} a=l \rightarrow \\ b=m \rightarrow \\ c=n \rightarrow \end{matrix}$ DALLA RETTA r

$$r: X + Z = Y - 2Z = 0$$

DI r VOGLIO l, m, n

$$\gamma = \begin{cases} X + Z = 0 \\ Y - 2Z = 0 \end{cases}$$

$$\begin{matrix} a & b & c \\ X + Z = 0 \end{matrix}$$

$$\begin{matrix} a' & b' & c' \\ Y - 2Z = 0 \end{matrix}$$

$$\begin{pmatrix} l & m & n \\ 1 & 0 & 1 \\ 0 & 1 & -2 \end{pmatrix}$$

$$l = \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} = -1$$

$$m = - \begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix} = +2$$

$$n = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$\begin{aligned} a &= l = -1 \\ b &= m = 2 \\ c &= n = 1 \end{aligned}$$

$$\pi : -X + 2Y + Z + d = 0$$

(2) DISTANZA $O(0,0,0)$ $\pi : -X + 2Y + Z + d = 0$

$$\frac{|0 + 0 + 0 + d|}{\sqrt{1 + 4 + 1}} = 2$$

$$\frac{|d|}{\sqrt{6}} = 2$$

$$|d| = 2\sqrt{6}$$

$$d = \pm 2\sqrt{6}$$

$$\pi_1 : -X + 2Y + Z + 2\sqrt{6} = 0$$

$$\pi_2 : -X + 2Y + Z - 2\sqrt{6} = 0$$

