

1) TROVARE UNA BASE PER OGNUNO DEGLI AUTOSPAZI

DELLA MATRICE

$$A = \begin{bmatrix} 0 & 10 & -8 \\ 0 & -2 & 0 \\ 0 & -6 & 0 \end{bmatrix}$$

UNO DEI DUE $\rightarrow \lambda = 0 \quad E(0) = \{(1, 0, 0)\}$
 $\rightarrow \lambda = 0 \rightarrow (1, 0, 0)$

$\lambda = -2 \quad E(-2) = \{(7, 1, 3)\}$
 $\lambda = -2 \rightarrow (7, 1, 3)$

$$A - \lambda I = \begin{bmatrix} -\lambda & 10 & -8 \\ 0 & -2-\lambda & 0 \\ 0 & -6 & -\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 10 & -8 \\ 0 & (-2-\lambda) & 0 \\ 0 & -6 & -\lambda \end{vmatrix} = \begin{vmatrix} -\lambda & 10 \\ 0 & (-2-\lambda) \\ 0 & -6 \end{vmatrix} = \lambda^2(-2-\lambda)$$

$\det(A - \lambda I) = 0 \quad \lambda^2(-2-\lambda) = 0 \rightarrow \begin{matrix} \lambda = 0 \\ \lambda = -2 \end{matrix}$ AUTOVALORI

$\lambda = 0 \quad A - 0I = \begin{pmatrix} 0 & 10 & -8 \\ 0 & -2 & 0 \\ 0 & -6 & 0 \end{pmatrix} \left[\begin{matrix} \text{PROPORZIONALI} \end{matrix} \right]$

$$\begin{pmatrix} 0 & 10 & -8 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} : 2 \\ : -2 \end{matrix} \rightarrow \begin{pmatrix} 0 & 5 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 5y - 4z = 0 \\ y = 0 \\ z = 0 \end{cases} \rightarrow \begin{cases} -4z = 0 \rightarrow z = 0 \\ y = 0 \\ x = t \end{cases} \quad (t, 0, 0) \rightarrow t(1, 0, 0)$$

$\lambda = -2 \quad A + 2I = \begin{pmatrix} 2 & 10 & -8 \\ 0 & 0 & 0 \\ 0 & -6 & 2 \end{pmatrix} \begin{matrix} : 2 \\ : 2 \end{matrix} \rightarrow \begin{pmatrix} 1 & 5 & -4 \\ 0 & 0 & 0 \\ 0 & -3 & 1 \end{pmatrix}$

$$\begin{pmatrix} \boxed{1} & 5 & -4 \\ 0 & \boxed{-3} & 1 \\ 0 & 0 & \boxed{0} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

"RIDOTTA"

$$\begin{cases} x + 5y - 4z = 0 \\ -3y + z = 0 \\ 0 = 0 \end{cases} \quad \begin{cases} x + 5y - 12y = 0 \\ z = 3y \\ 0 = 0 \end{cases} \quad \begin{cases} x = 7y \\ z = 3y \\ y = t \end{cases}$$

$$\begin{cases} x = 7t \\ y = t \\ z = 3t \end{cases} \quad t(7, 1, 3)$$

2) TROVARE LA SOLUZIONE GENERALE DEL SISTEMA LINEARE

$$3x - y + 9z - 25 = x - y + 7z - 9 = 2x + y - 4z - 15 = 0$$

1° POSSIBILITÀ $(8-t; 6t-1; t)$ OPPURE 2° POSS. $(8, -1; 0) + t(-1; 6; 1)$

3° POSS. $(8-z; 6t-1; z)$

$$\begin{cases} 3x - y + 9z = 25 \\ x - y + 7z = 9 \\ 2x + y - 4z = 15 \end{cases}$$

SE VOLETE

- 1) SOSTITUZIONE
- 2) CRAMER
- 3) RIDUZIONE

RIDUZIONE

$$\begin{pmatrix} \boxed{3} & -1 & 9 & : & 25 \\ 1 & -1 & 7 & : & 9 \\ 2 & 1 & -4 & : & 15 \end{pmatrix} \quad r_2 \rightarrow -3r_2 + r_1$$

$$\begin{array}{cccc|c} & -3r_2 & -3 & +3 & -21 & -27 \\ & r_1 & 3 & -1 & 9 & 25 \\ \hline & & 0 & 2 & -12 & -2 \end{array}$$

$$\begin{array}{l} 6 \quad \xrightarrow{-2} \\ -6 \quad \xrightarrow{-3} \end{array} \begin{pmatrix} \boxed{3} & -1 & 9 & : & 25 \\ 0 & 2 & -12 & : & -2 \\ 2 & 1 & -4 & : & 15 \end{pmatrix} \quad r_3 \rightarrow -3r_3 + 2r_1$$

$$\begin{array}{cccc|c} & -3r_3 & -6 & -3 & +12 & -45 \\ & 2r_1 & 6 & -2 & 18 & 50 \\ \hline & & 0 & -5 & 30 & 5 \end{array}$$

$$\begin{pmatrix} 3 & -1 & 9 & : & 25 \\ 0 & 2 & -12 & : & -2 \end{pmatrix} \quad : 2$$

$$\left(\begin{array}{ccc|c} 0 & 2 & -12 & -2 \\ 0 & -5 & 30 & 5 \end{array} \right) \begin{array}{l} : 2 \\ : 5 \end{array}$$

$$0 \quad -5 \quad 30 \quad 5$$

$$\left(\begin{array}{ccc|c} 3 & -1 & 9 & 25 \\ 0 & 1 & -6 & -1 \\ 0 & -1 & 6 & 1 \end{array} \right) \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{OPPOSITE}$$

$$\left(\begin{array}{ccc|c} \boxed{3} & -1 & 9 & 25 \\ 0 & \boxed{1} & -6 & -1 \\ 0 & 0 & \boxed{0} & 0 \end{array} \right) \text{RIDOTTA}$$

$$\begin{cases} 3x - y + 9z = 25 \\ y - 6z = -1 \\ 0 = 0 \end{cases}$$

$$\begin{cases} 3x - 6z + 1 + 9z = 25 \\ \rightarrow y = 6z - 1 \end{cases}$$

$$\begin{cases} 3x + 3z = 24 : (3) & x + z = 8 \rightarrow x = 8 - z \\ & y = 6z - 1 \\ & z = t \end{cases}$$

$$(8 - t; 6t - 1; t) \quad \begin{array}{l} \text{SEPARO LE } t \text{ DAI} \\ \text{TERMINI NOTI} \end{array}$$

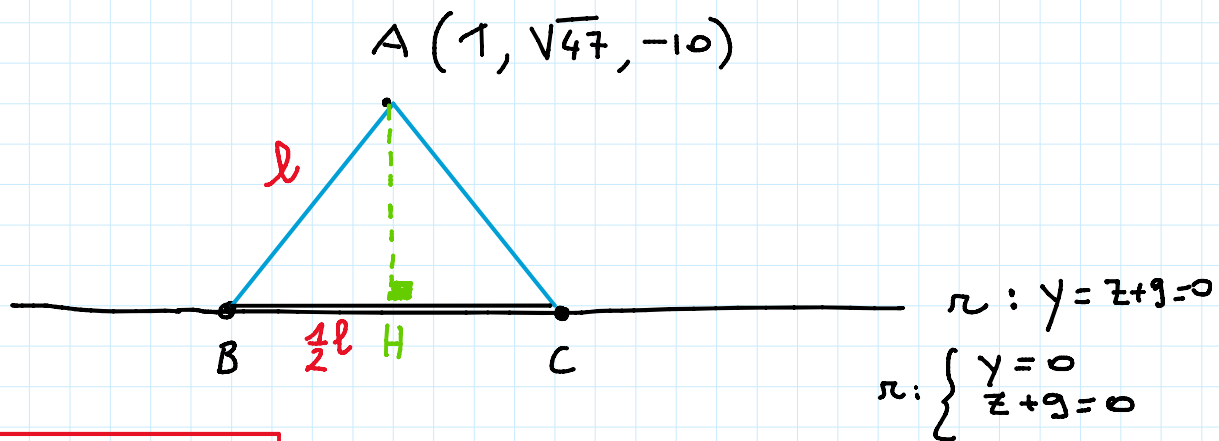
$$(8; -1; 0) + (-t, 6t, t)$$

$$(8; -1; 0) + t(-1, 6, 1)$$

3) SIA $A(1; \sqrt{47}; -10)$. SULLA RETTA $\pi: y = z + 9 = 0$
TROVA DUE PUNTI B e C TALI CHE IL TRIANGOLO
ABC SIA EQUILATERO

$$(-3; 0; -9)$$

$$(5, 0, -9)$$



$$BH = \frac{1}{2} AB$$

USERO QUESTA

A CE L'HO

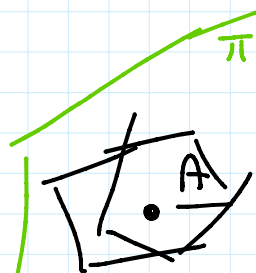
B E RETTA $\pi \Rightarrow$ PARAMETRICA

H MI SERVE

$$\begin{cases} y = 0 \\ z + 9 = 0 \end{cases} \quad \begin{cases} x = t \\ y = 0 \\ z = -9 \end{cases}$$

GENERIC $B(t, 0, -9)$

CERCO H



STELLA DI PIANI. TRA ESSI, PRENDI IL PIANO $\perp$ AD r

$$\pi \quad \begin{cases} x = t & l = 1 \\ y = 0 & m = 0 \\ z = -9 & n = 0 \end{cases}$$

π $\rightarrow$ PASSA PER $A(1, \sqrt{47}, -10)$

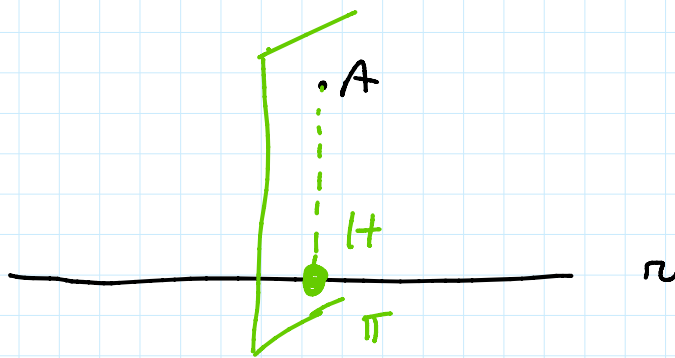
$\perp r$

$$\begin{aligned} a = l &\rightarrow a = 1 \\ b = m &\rightarrow b = 0 \\ c = n &\rightarrow c = 0 \end{aligned}$$

$$\pi: a(x - x_A) + b(y - y_A) + c(z - z_A) = 0$$

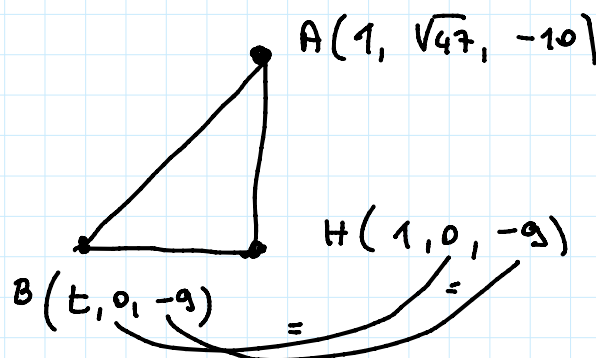
$$1(x - 1) + 0(y - \sqrt{47}) + 0(z + 10) = 0$$

$$\boxed{\pi : x-1=0}$$

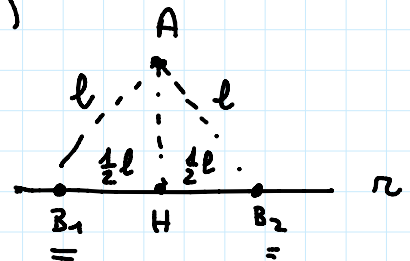


$$\left\{ \begin{array}{l} \pi \\ z \end{array} \right\} \rightarrow H$$

$$H = \left\{ \begin{array}{l} x-1=0 \\ \left\{ \begin{array}{l} x=t \\ y=0 \\ z=-9 \end{array} \right. \end{array} \right. \quad \left\{ \begin{array}{l} x=1 \\ y=0 \\ z=-9 \end{array} \right. \quad H(1, 0, -9)$$



$$BH = \frac{1}{2} AB \quad \text{SCOPRIAMO B}$$



$$BH = |x_B - x_H| = |t-1| \quad \left(\sqrt{(t-1)^2 + (0-\sqrt{47})^2 + (-9+10)^2} \right)$$

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

$$AB = \sqrt{(t-1)^2 + (0-\sqrt{47})^2 + (-9+10)^2} = \sqrt{t^2 - 2t + 1 + 47 + 1}$$

$$AB = \sqrt{t^2 - 2t + 49}$$

$$\text{USO LA RELAZIONE } BH = \frac{1}{2} AB$$

$$\left(|t-1| \right)^2 = \left(\frac{1}{2} \sqrt{t^2 - 2t + 49} \right)^2$$

$$1^2 \dots \dots \dots 1^2$$

$$t^2 - 2t + 1 = \frac{1}{4} (t^2 - 2t + 49) \quad \text{m.c.m.}$$

$$4t^2 - 8t + 4 = t^2 - 2t + 49$$

$$3t^2 - 6t - 45 = 0 \quad (:3)$$

$$t^2 - 2t - 15 = 0 \quad (\text{uso } \Delta)$$

$$\text{oppure } (t-5)(t+3) = 0 \quad \begin{cases} t=5 \\ t=-3 \end{cases}$$

$$(5, 0, -9) \quad (-3, 0, -9)$$

4) SIANO $A(3, 0, 0)$ $B(0, 2, 0)$ $C(0, 0, t)$

TROVA I VALORI DI t PER CUI IL PIANO
PASSANTE PER A, B, C FORMA CON IL PIANO xy UN
ANGOLO DI $\frac{\pi}{6}$ RADIANTI

NON LO FATE

$$t = \sqrt{\frac{12}{13}}$$

$$t = \pm \frac{\sqrt{4 \cdot 3}}{\sqrt{13}} = \pm \frac{2\sqrt{3}}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \pm \frac{2\sqrt{39}}{13}$$

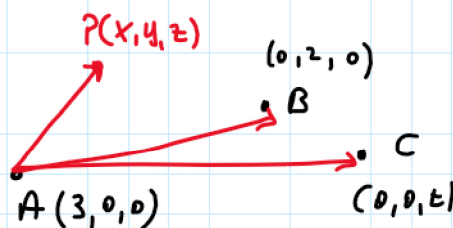
PIANO A, B, C

1° MODO

$$\begin{vmatrix} x & y & z & 1 \\ x_A & y_A & z_A & 1 \\ x_B & y_B & z_B & 1 \\ x_C & y_C & z_C & 1 \end{vmatrix} = 0$$

LAPLACE

2° MODO



3 VETTORI SONO
COMPLANARI SE SONO

L.D. $\Rightarrow \det A = 0$

$$\vec{AP} = (3-x; 0-y; 0-z) = (3-x; -y; -z)$$

$$\vec{AB} = (3-0; 0-2; 0-0) = (3; -2; 0)$$

$$A_P = (3-x; 0-y; 0-z) = (3-x; -y, -z)$$

$$A_B = (3-0; 0-2; 0-0) = (3; -2; 0)$$

$$A_C = (3-0; 0-0; 0-t) = (3; 0; -t)$$

PIANO PASSANTE PER A, B e C

$$\begin{vmatrix} (3-x) & -y & -z \\ 3 & -2 & 0 \\ 3 & 0 & -t \end{vmatrix} = 0$$

$$\begin{vmatrix} (3-x) & -y & -z \\ 3 & -2 & 0 \\ 3 & 0 & -t \end{vmatrix} \begin{vmatrix} (3-x) & -y \\ 3 & -2 \\ 3 & 0 \end{vmatrix} = 0$$

$$2t(3-x) - 6z - 3ty = 0 \quad \text{CAMBIO SEGNO}$$

$$-2t(3-x) + 6z + 3ty = 0$$

$$-6t + 2tx + 6z + 3ty = 0$$

$$\text{II} \quad \underbrace{2t}_{a'}x + \underbrace{3t}_{b'}y + \underbrace{6}_{c'}z - 6t = 0 \quad \text{PIANO } xy$$

$$1z = 0 \quad a'=0 \quad b'=0 \quad c'=1$$

$$\alpha = \frac{\pi}{6}$$

$$\cos \frac{\pi}{6} = \pm \frac{aa' + bb' + cc'}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{a'^2 + b'^2 + c'^2}}$$

$$\frac{\sqrt{3}}{2} = \pm \frac{6}{\sqrt{4t^2 + 9t^2 + 36} \cdot \sqrt{1}} \quad ()^2 = ()^2$$

$$\frac{3}{4} = + \frac{36}{4t^2 + 9t^2 + 36} \quad \text{m. c. m.}$$

$$\underline{12t^2} + \underline{27t^2} + \underline{108} = \underline{144}$$

$$39t^2 = 36$$

$$t^2 = \frac{36}{39}$$

$$t^2 = \frac{12}{13} \quad t = \pm \sqrt{\frac{12}{13}}$$

5) SE ESISTE UNA CONICA PASSANTE PER I PUNTI

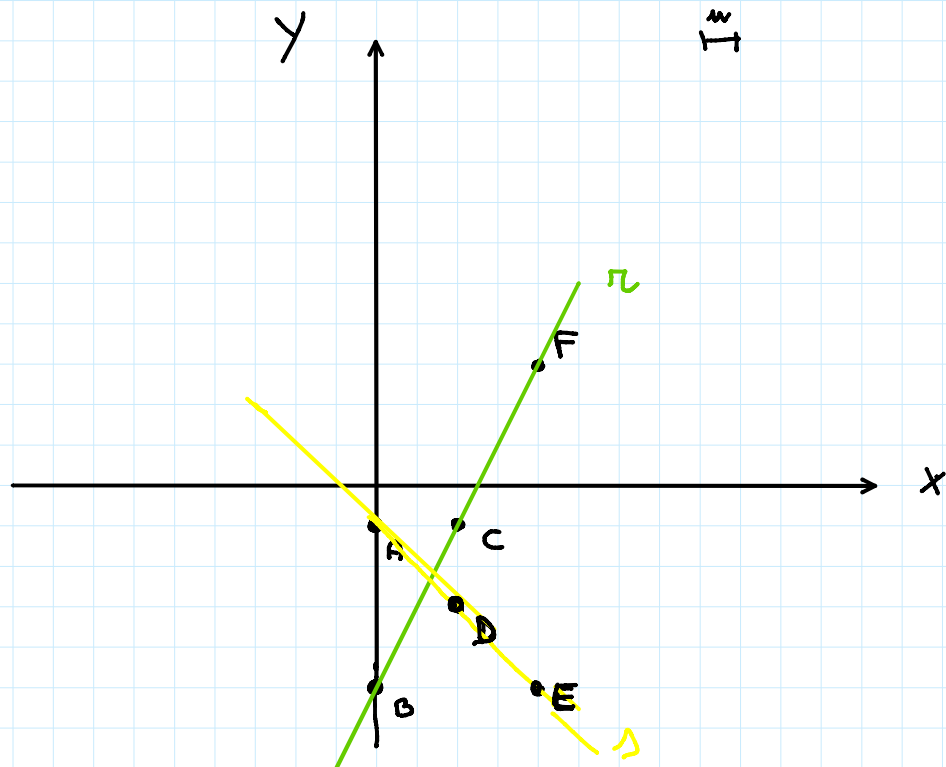
A(0; -1) B(0; -5); C(2; -1) D(2; -3)

E(4; -5) F(4; 3)

ALLORA SCRIVERE L'EQUAZIONE. ALTRIMENTI DIRE PERCHÉ NON ESISTE

$$(x+y+1)(2x-y-5) = 0 \quad \text{OPPURE} \quad 2x^2 + xy - y^2 - 3x - 6y - 5 = 0$$

DISEGNO



CONICHE DEGENERI NEL PRODOTTO DI 2 RETTE

I PUNTI, PER DARE UNA CONICA, DEVONO ESSERE **TUTTI**

CONTENUTI IN 2 RETTE. SE ANCHE UNO RESTA FUORI **NON È**
UNA CONICA

CONICA (EQUAZIONE DI π) $\cdot$ (EQUAZIONE DI Δ) = 0

$\pi \begin{cases} B(0, -5) \\ C(2, -1) \\ F(4, 3) \end{cases} \rightarrow \text{uso}$

$y = mx + q$

$B \begin{cases} -5 = m \cdot 0 + q \\ C \begin{cases} -1 = m \cdot 2 + q \end{cases}$

$\begin{cases} q = -5 \\ 2m - 5 = -1 \quad 2m = 4 \quad m = 2 \end{cases}$

$\pi: y = 2x - 5$

$2x - y - 5 = 0$

CONTROLO SE
ANCHE $F \in \pi$

$F(4, 3) \quad 2x - y - 5 = 0$

$8 - 3 - 5 = 0$
 $0 = 0$ SI

$\Delta \begin{cases} A(0, -1) \\ D(2, -3) \\ E(4, -5) \end{cases} \rightarrow \text{uso}$

$y = mx + q$

$A \begin{cases} -1 = m \cdot 0 + q \\ D \begin{cases} -3 = m \cdot 2 + q \end{cases} \end{cases} \begin{cases} q = -1 \\ 2m - 1 = -3 \end{cases}$

$\begin{cases} q = -1 \\ 2m = -2 \quad m = -1 \end{cases}$

$y = -x - 1 \quad x + y + 1 = 0 \quad \Delta$

CONTROLO SE $E(4, -5) \in \Delta$

$4 - 5 + 1 = 0 \quad 0 = 0$ SI

CONICA $(x + y + 1) \cdot (2x - y - 5) = 0$

$2x^2 - xy - 5x + 2xy - y^2 - 5y + 2x - y - 5 = 0$

$2x^2 + xy - y^2 - 3x - 6y - 5 = 0$

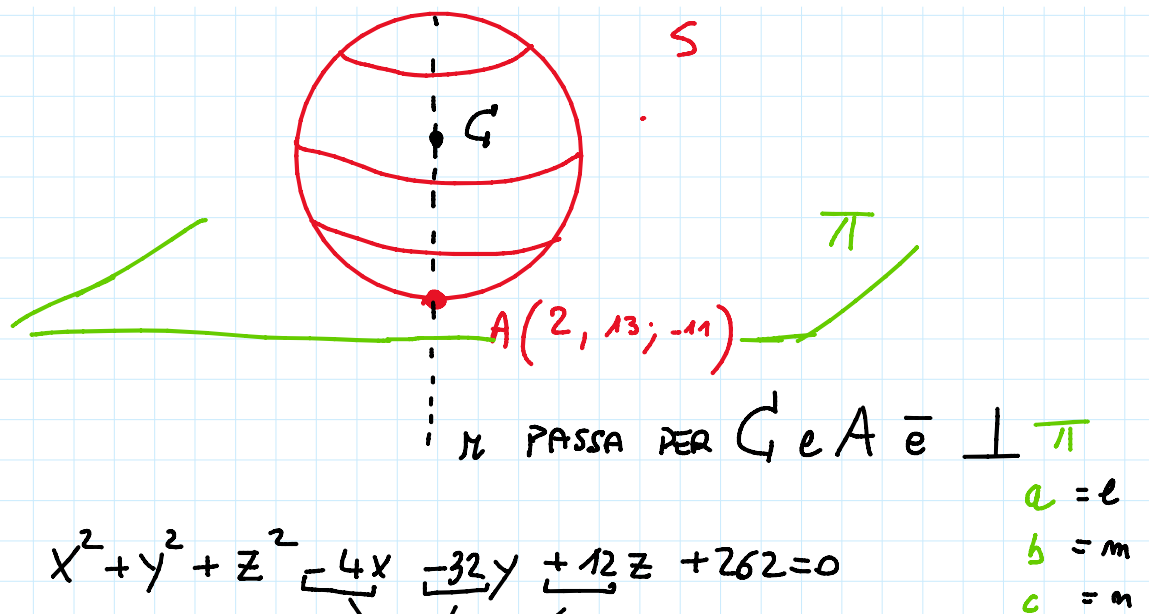
6) SIA S LA SFERA DI EQUAZIONE $x^2 + y^2 + z^2 - 4x - 32y + 12z + 262 = 0$

SCRIVI L'EQ. DEL PIANO TANGENTE AD S NEL SUO

PUNTO $A(2; 13; -11)$

$3y + 5z + 16 = 0$

:



$$X^2 + Y^2 + Z^2 - 4X - 32Y + 12Z + 262 = 0$$

CAMBIO SEGNO $\div 2$

$$C(2, 16, -6)$$

$$A(2, 13, -11)$$

r PASSA PER C e A

$$\begin{aligned} l &= x_C - x_A = 0 \\ m &= y_C - y_A = +3 \\ n &= z_C - z_A = +5 \end{aligned}$$

$$\pi \perp r \Rightarrow \begin{aligned} a &= l = 0 \\ b &= m = +3 \\ c &= n = +5 \end{aligned}$$

π passa per $A(2, 13, -11)$

$$\pi: a(x - x_A) + b(y - y_C) + c(z - z_C) = 0$$

$$0(x - 2) + 3(y - 13) + 5(z + 11) = 0$$

$$3y - 39 + 5z + 55 = 0$$

$$\pi \mid 3y + 5z + 16 = 0$$