

$$a_n = 2 + \frac{1}{n} \quad n \in \mathbb{N}_0$$

PROVE $a_1 = 2 + 1 = 3$

$$a_2 = 2 + \frac{1}{2} = 2,5$$

$$a_3 = 2 + \frac{1}{3} = 2,3 \quad \text{BASTA}$$

II DECRESCERE $\rightarrow$ QUELLO DI PRIMA È
PIÙ GRANDE DI
QUELLO DI QUELLO DI DOPO "

$$a_{(m+1)}$$

$$a_m > a_{(m+1)} \quad \forall m$$

$$\cancel{2} + \frac{1}{m} > \cancel{2} + \frac{1}{(m+1)}$$

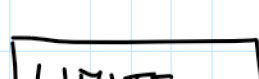
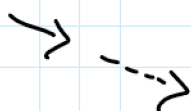
$$\frac{1}{m} > \frac{1}{(m+1)}$$

$$\Downarrow$$

$$\cancel{m} < \cancel{m+1}$$

$$0 < 1 \quad \text{VERA} \quad \forall m$$

II $a_1 = 3$ "IL PIÙ GRANDE"



(a_n)

$\nearrow$ LIMITE $\lim_{n \rightarrow +\infty} \left(2 + \frac{1}{n}\right) = 2$
 "IL PIÙ PICCOLO"

$$a_1 = 3 \quad \text{MAX}$$

2 ESTREMO INFERIORE INF.

$$a_n = \frac{(-1)^n n + 2}{n-1}$$

$$n \in \mathbb{N} \setminus \{1\}$$

$$n \text{ PARI } (-1)^n = 1$$

$$a_n = \frac{n+2}{n-1}$$

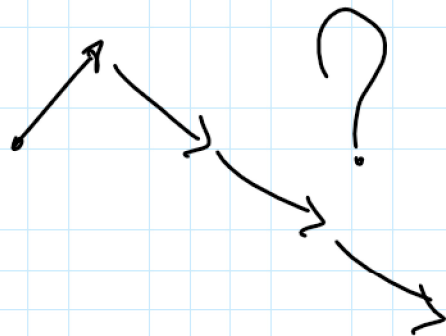
"PROVE"

$$a_0 = -2$$

$$a_2 = 4$$

$$a_4 = \frac{6}{3} = 2$$

$$a_6 = \frac{8}{5} = 1.6$$



$$a_n > a_{(n+2)}$$

$$\frac{n+2}{n-1} > \frac{(n+2)+2}{(n+2)-1}$$

n PARI $2K$

$$a_{2K} > a_{(2K+2)}$$

$$\frac{2K+2}{2K-1} > \frac{(2K+2)+2}{(2K+2)-1}$$

$$\left(\frac{m+2}{m-1} > \frac{(m+2)+2}{(m+2)-1} \right) \quad \left| \quad \frac{2k+2}{2k-1} > \frac{(2k+2)+2}{(2k+2)-1} \right.$$

$$\frac{m+2}{m-1} > \frac{m+4}{m+1}$$

$$\frac{m+2}{m-1} - \frac{m+4}{m+1} > 0$$

$$\frac{(m+2)(m+1) - (m+4)(m-1)}{(m-1)(m+1)} > 0$$

↑
E' POSITIVO
PER

$$\frac{m^2 + m + 2m + 2 - (m^2 - m + 4m - 4)}{(m-1)} > 0$$

$$\frac{m^2 + 3m + 2 - m^2 + m - 4m + 4}{m-1} > 0$$

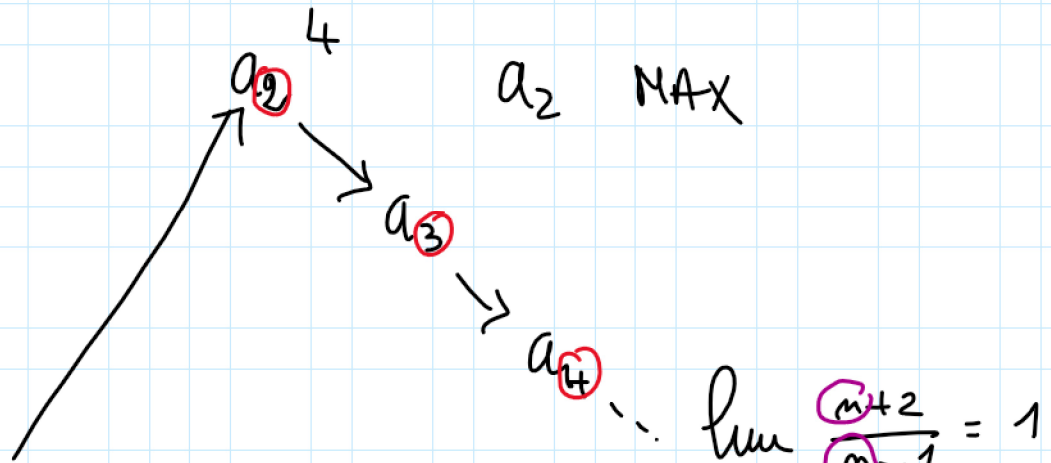
$$\frac{6}{m-1} > 0 \quad \text{FRATTA}$$

N 6 SEMPRE POSITIVO
D $m-1 \geq 0 \quad m \geq 1$

	1					
N	+	+	+			
D	-	-	+			
N/D	-		+			

$$m > 1$$

a_m DECRESCe QUANDO $m > 1$



$$a_0 = -2 \text{ MIN}$$

$$\lim_{n \rightarrow \infty} \frac{n+2}{n-1} = 1$$

$$n \text{ dispari } (-1)^n = -1$$

$$a_n = \frac{-n+2}{n-1}$$

$$a_3 = -\frac{1}{2}$$

$$a_5 = -\frac{3}{4}$$

$$a_7 = -\frac{5}{6}$$

$$a_n > a_{n+2}$$

$$\frac{-n+2}{n-1} > \frac{-(n+2)+2}{(n+2)-1}$$

$$\frac{-n+2}{n-1} > \frac{-n-2+2}{n+1}$$

$$\frac{-n+2}{n-1} + \frac{n}{n+1} > 0$$

$$\frac{(-n+2)(n+1) + n(n-1)}{(n-1)(n+1)} > 0$$

È POSITIVO $\frac{(-n+2)(n+1)}{(n-1)(n+1)}$ È POSITIVO

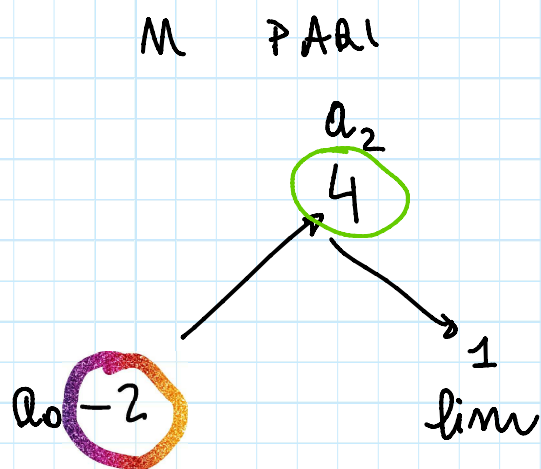
$n \in \{3, 5, 7, \dots\}$

$$-n^2 - n + 2n + 2 + n^2 - n > 0 \text{ VERA}$$

a_n decresce

$$a_3 = -\frac{1}{2} \text{ MAX}$$





L MAX
-2 MIN

$$\lim_{n \rightarrow +\infty} \frac{-n+2}{n-1} = -1 \quad \text{INF}$$

n DISPARI

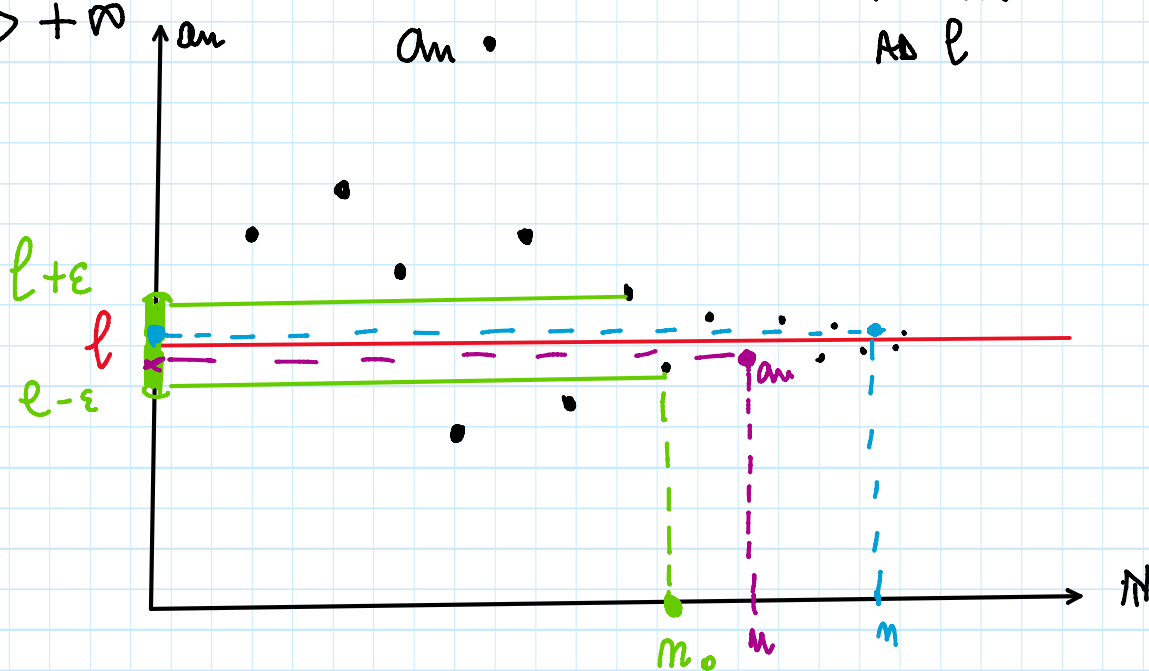
$$a_3 = -\frac{1}{2}$$

$\rightarrow -1$
lim

$$\lim_{n \rightarrow +\infty} a_n = l$$

a_n

VUOL DIRE: PIÙ n CRESCE
PIÙ a_n SI AVVICINA
AD l



$$\forall \epsilon > 0 \quad \exists n_0 \quad | \quad \text{se } n > n_0 \quad \underbrace{l-\epsilon < a_n < l+\epsilon}$$

$n > n_0$ $n > n_0$

$$\forall \varepsilon > 0 \quad \exists m_0 \mid \text{se } m > m_0 \quad \underbrace{l - \varepsilon < a_m < l + \varepsilon}_{\text{cioè}} \mid a_m - l < \varepsilon$$

VERIFICARE CHE

$$\lim_{m \rightarrow +\infty} \frac{a_m}{m+2} = 1$$

PARTO DA $\mid a_m - l \mid < \varepsilon$

E CERCO IL MIO m_0 CHE VERIFICA IL LIMITE

$$\left| \frac{m+2}{m-1} - 1 \right| < \varepsilon$$

$$\left| \frac{m+2 - (m-1)}{m-1} \right| < \varepsilon$$

$$\left| \frac{\cancel{m+2} - \cancel{m} + 1}{m-1} \right| < \varepsilon$$

$$\left| \frac{3}{m-1} \right| < \varepsilon$$

$$-\varepsilon < \frac{3}{m-1} < \varepsilon$$

$$A \quad \left\{ \begin{array}{l} \frac{3}{m-1} < \varepsilon \end{array} \right.$$

$$B \quad \left\{ \begin{array}{l} \frac{3}{m-1} > -\varepsilon \end{array} \right.$$

$$\begin{array}{l} \mid f(x) \mid < K \quad K \in \mathbb{R}^+ \\ \underbrace{-K \quad K} \\ -K < f(x) < K \\ \downarrow \\ \left\{ \begin{array}{l} f(x) < K \\ f(x) > -K \end{array} \right. \end{array}$$

se $m=0$ LA (B) FALSA

$$\frac{3}{-1} > -\varepsilon \quad -3 > -\varepsilon$$

se $m=1$ NO

se $m \geq 2$ (B) VERA

$$A \quad \frac{3}{m-1} < \varepsilon \quad \left[3 \cdot \frac{m-1}{3} \right] \quad \frac{1}{\varepsilon} \cdot 3$$

$$m-1 > \frac{3}{\varepsilon} \quad m > \boxed{1 + \frac{3}{\varepsilon}}$$

$$\text{Se } m > m_0 \quad \left| \frac{m+2}{m-1} - 1 \right| < \varepsilon \quad \text{cioè } |a_n - l| < \varepsilon$$

m_0

$$\lim_{n \rightarrow +\infty} \frac{m+2}{m-1} = 1$$

PER VOI.

DIMOSTRA USANDO LA DEFINIZIONE

$$\lim_{n \rightarrow +\infty} \frac{n+2}{n^2+3n} = 0$$

SUGG. RICORDA CHE $m \in \mathbb{N}_0$
QUINDI TROVA LE
QUANTITÀ POSITIVE

$$\lim_{n \rightarrow +\infty} \frac{n+1}{2n+3} = \frac{1}{2}$$

$$\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{2^n} \right) = 1$$