

DEFINIZIONE DI DOMINIO:

SONO I VALORI CHE PUÒ ASSUMERE LA x AFFINCHÉ LA FUNZIONE (LA y) SIA REALE E DEFINITA

1) FUNZIONE FRATTA : $\frac{1}{0}$ NON È DEFINITO

DENOMINATORE $\neq 0$

2) LOGARITMO $\log(\bar{-})$ o $\log(\downarrow 0)$ NON È DEFINITO

ARGOMENTO > 0

3) RADICE DI INDICE PARI $\sqrt[2m]{\bar{-}}$ NON È REALE

RADICANDO ≥ 0

4) $y = (f(x))^{g(x)}$ $(\bar{-})^{g(x)}$ NON È DEFINITO

BASE $f(x) > 0$

5) $\arcsin f(x)$ oppure $\arccos f(x)$

$-1 \leq f(x) \leq 1$

LE FUNZIONI CHE NON RIENTRANO IN QUESTI 5 CASI HANNO $D: \mathbb{R}$

ESERCIZI

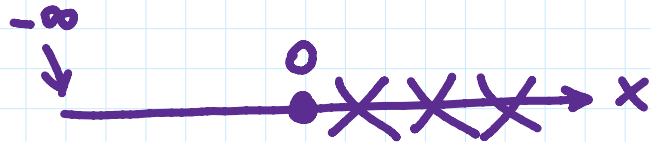
ESERCIZI

$$f(x) = \sqrt{e^{2x} - e^{3x}}$$

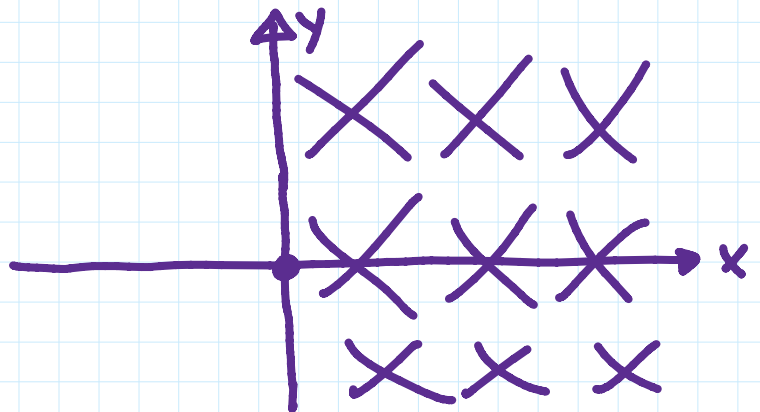
$$\triangleright e^{2x} - e^{3x} \geq 0 \quad e^{2x} \geq e^{3x} \quad e = 2,7 > 1$$

$$\text{NON CAMBIO VERSO} \quad 2x \geq 3x \quad -x \geq 0 \quad x \leq 0$$

DISEGNINO



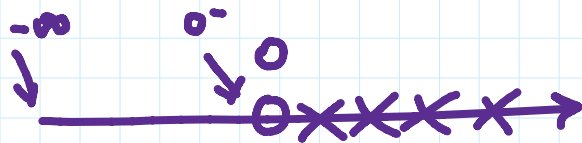
DISEGNONE



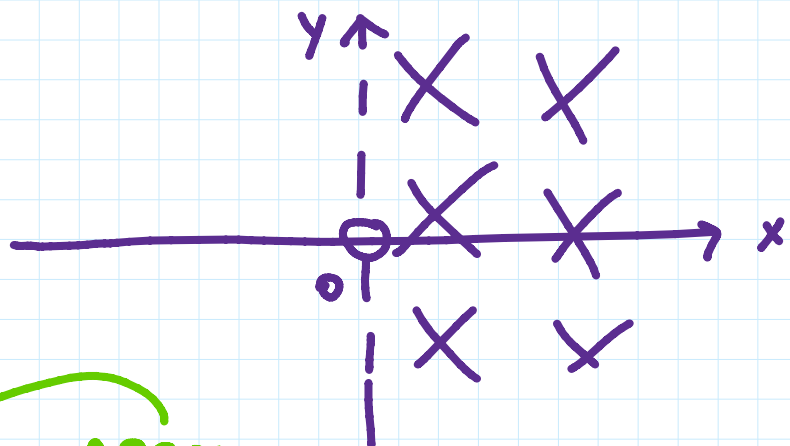
$$f(x) = \ln(e^{2x} - e^{3x})$$

$$\triangleright e^{2x} - e^{3x} > 0 \quad x < 0$$

DISEGNINO



DISEGNONE



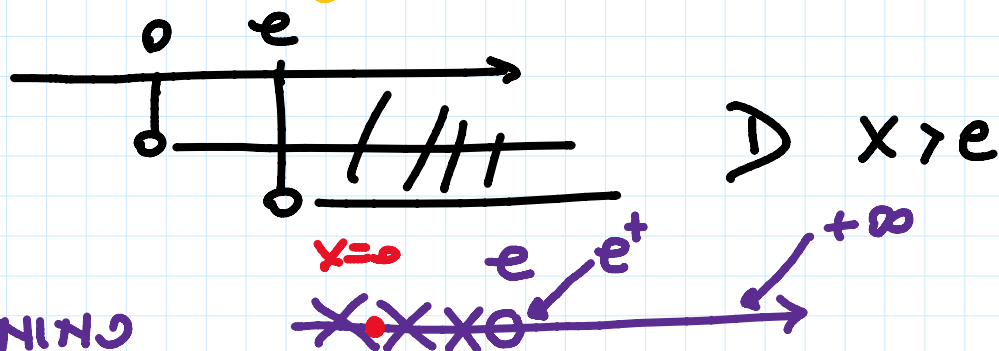
ARGOM

$$f(x) = \ln(\ln x - 1)$$

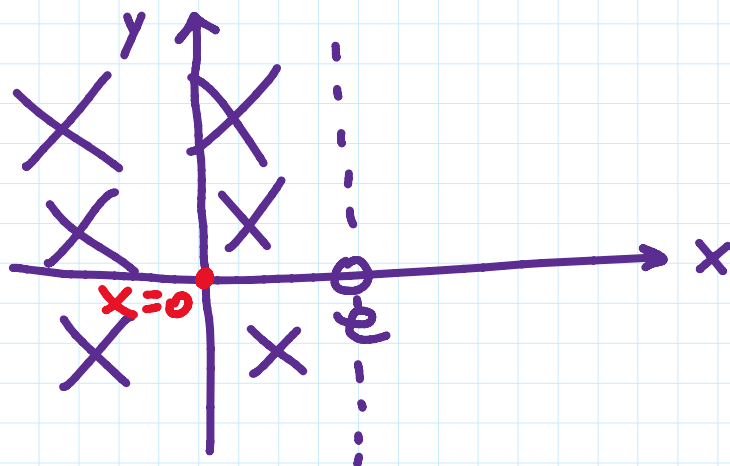
ARGOMENTO

"GIF"

$$\begin{cases} x > 0 \\ \ln x - 1 > 0 \rightarrow \ln x > 1 \quad x > e^1 \quad x > e \end{cases}$$



DISegnare



$$f(x) = \frac{\ln |e^x - 1|}{e^{x^2+4x-5} - 1}$$

Dominio

$$\begin{cases} A \{ |e^x - 1| > 0 \\ B \{ e^{x^2+4x-5} - 1 \neq 0 \end{cases}$$

"GIF"

$|e^x - 1| > 0 \rightarrow + > 0$ OK

$\Delta \quad |e^x - 1| = 0 \quad \forall x \in \mathbb{R} \quad x \neq 0$

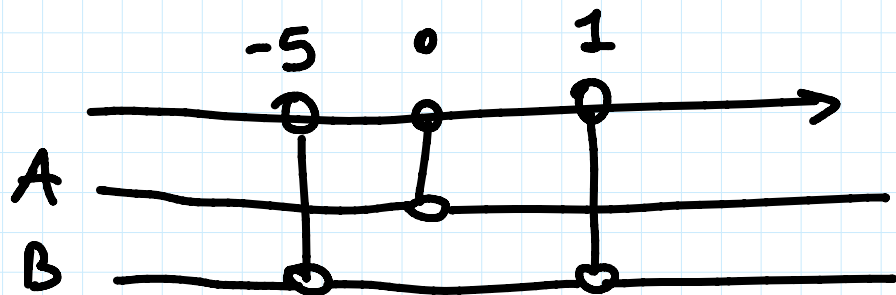
A $|e^x - 1| > 0 \quad \forall x \in \mathbb{R}, x \neq 0$

Red annotations: $\rightarrow + > 0 \text{ OK}$
 $\hookrightarrow 0 > 0 \text{ NO} \rightarrow e^x - 1 \neq 0 \quad e^x \neq 1$
 $e^x \neq e^0 \quad x \neq 0$

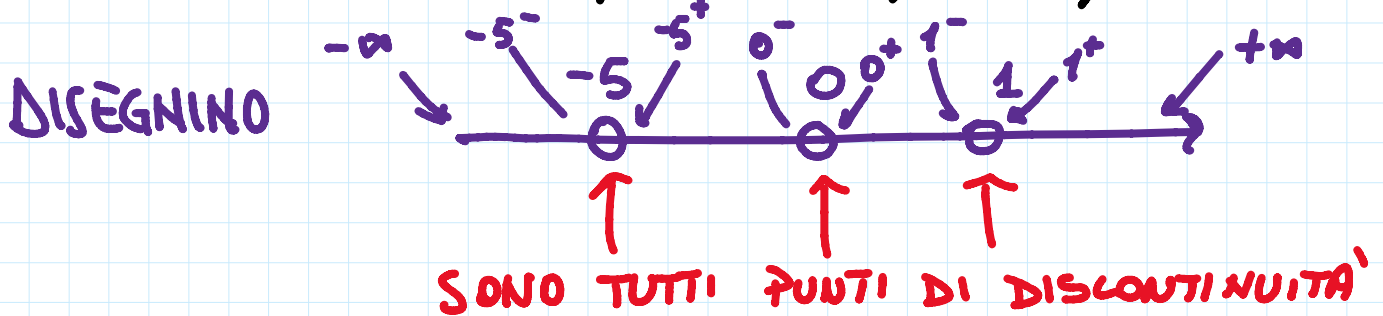
B $e^{x^2+4x-5} - 1 \neq 0 \quad e^{x^2+4x-5} \neq 1$

$x^2 + 4x - 5 = 0 \quad x \neq 1$
 $x \neq -5$

GIF



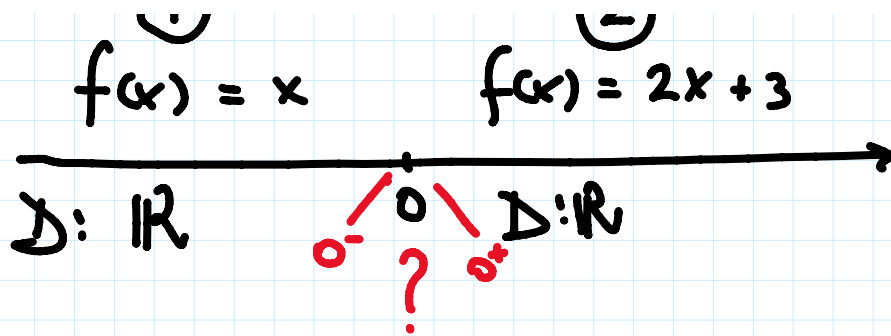
D $\forall x \in \mathbb{R}, x \neq -5; x \neq 0; x \neq 1$



PUNTI DI DISCONTINUITA'

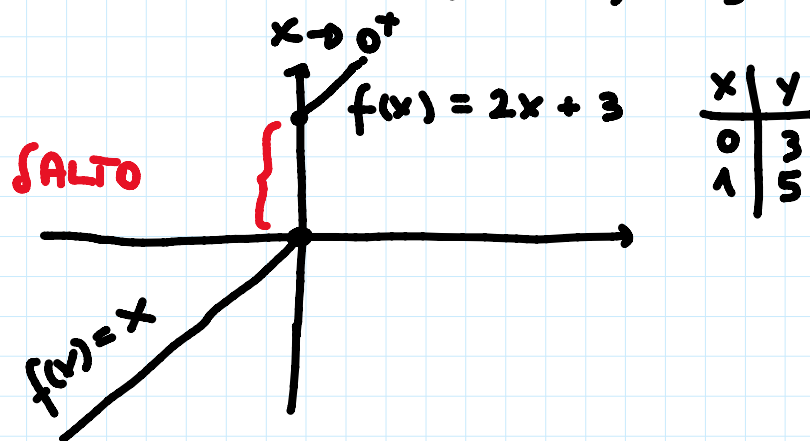
$f(x) = \begin{cases} \textcircled{1} & x & \text{SE} & x < 0 \\ \textcircled{2} & 2x + 3 & \text{SE} & x \geq 0 \end{cases}$ FUNZIONE DEFINITA PER CASI

$\textcircled{1} \quad f(x) = x \quad \textcircled{2} \quad f(x) = 2x + 3$



$$\lim_{x \rightarrow 0^-} x = 0$$

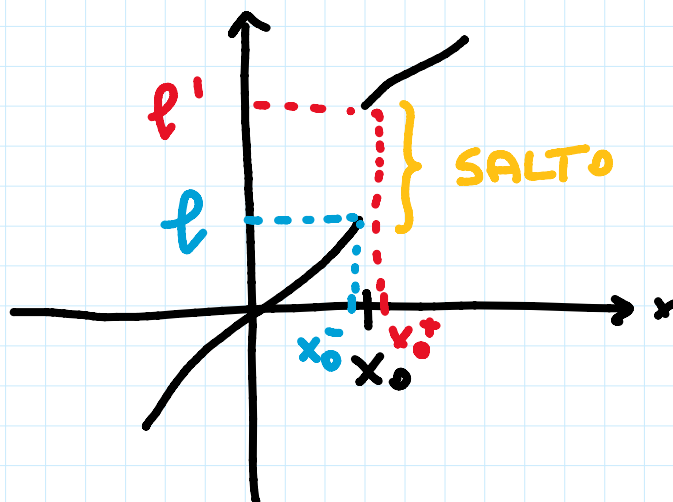
$$\lim_{x \rightarrow 0^+} (2x + 3) = 3$$



x_0 È UNA DISCONTINUITÀ DI 1^a SPECIE SE

$$\lim_{x \rightarrow x_0^-} f(x) \text{ E } \lim_{x \rightarrow x_0^+} f(x) \text{ FINITI E DIVERSI}$$

$$l \text{ E } l' \Rightarrow \text{SALTO} \quad |l - l'|$$

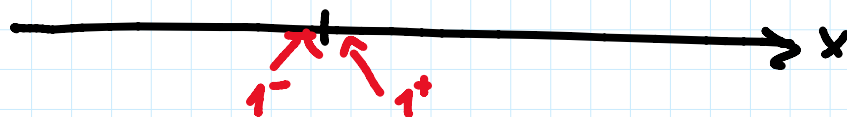


ESATTE

DATA $f(x) = \begin{cases} -2x + a & \text{se } x < 1 \\ x + 5 & \text{se } x \geq 1 \end{cases}$

TROVA IL VALORE DI a IN MODO CHE IN $x=1$
 $f(x)$ PRESENTI UN SALTO $= 5$

$$f(x) = -2x + a \quad f(x) = x + 5$$



$$\lim_{x \rightarrow 1^-} (-2x + a) = -2(1) + a = -2 + a = \ell$$

$$\lim_{x \rightarrow 1^+} (x + 5) = 1 + 5 = 6 = \ell'$$

$$\text{SALTO } |\ell - \ell'| = |-2 + a - 6|$$

$$\text{SALTO } |a - 8|$$

$$|a - 8| = 5$$

$$a - 8 = -5$$

$$a = 3$$

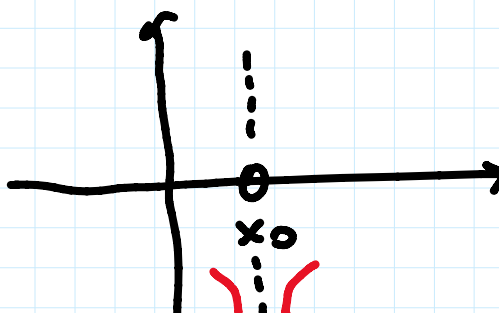
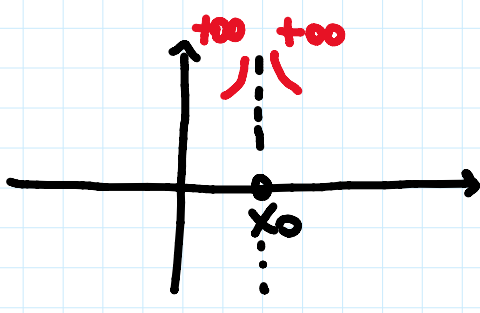
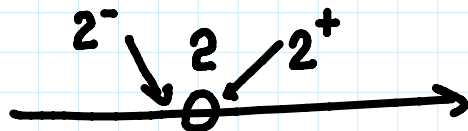
$$a - 8 = +5$$

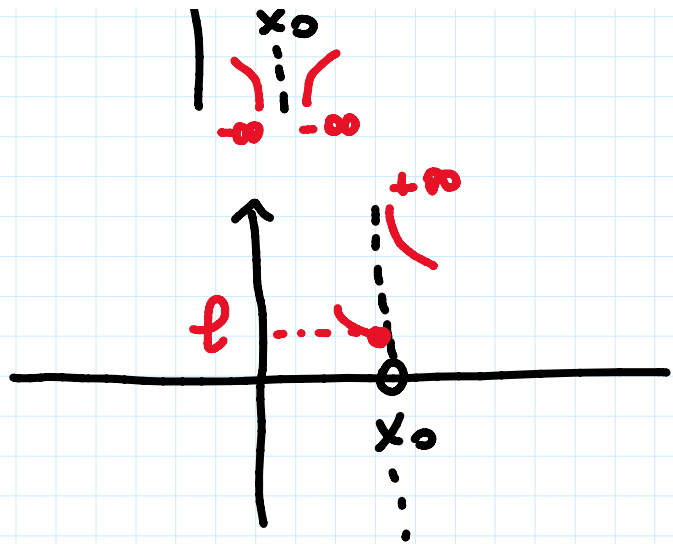
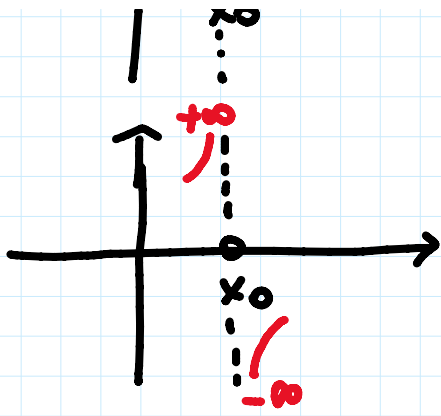
$$a = 13$$

DISCONTINUITÀ 2^a SPECIE

VIENE FUORI DALLO STUDIO DEL DOMINIO

ES. DOMINIO $x \neq 2$





$X = x_0$ ASINTOTO VERTICALE DX

$$f(x) = e^{\frac{1}{x-1}}$$

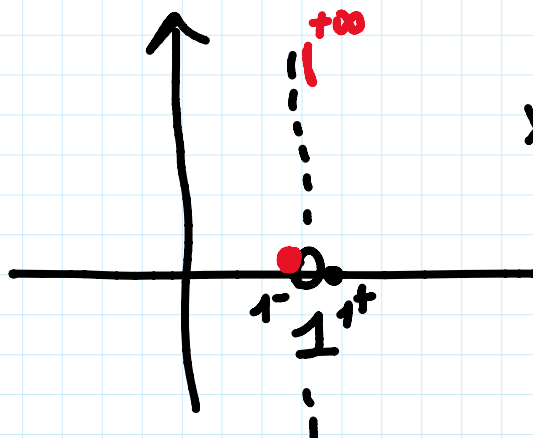
$D: x-1 \neq 0 \quad x \neq 1$

$$\lim_{x \rightarrow 1^+} e^{\frac{1}{x-1}} = e^{\frac{1}{1-1}} = e^{\frac{1}{0^+}} = e^{+\infty} = +\infty$$

$\frac{1}{0} \rightarrow$ QUANDO HO 0 DEVO CAPIRE SE È 0^+ OPPURE 0^- LO CAPISCO DA $x \rightarrow 1^+ \quad 1^+ \sim 1,1$
 $x-1 \sim 1,1-1 = 0^+$

$$\lim_{x \rightarrow 1^-} e^{\frac{1}{x-1}} = e^{\frac{1}{1-1}} = e^{\frac{1}{0^-}} = e^{-\infty} = 0$$

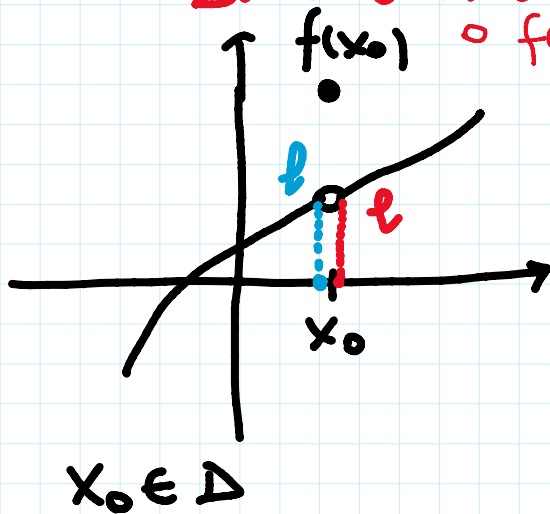
$1^- \sim 0,9$



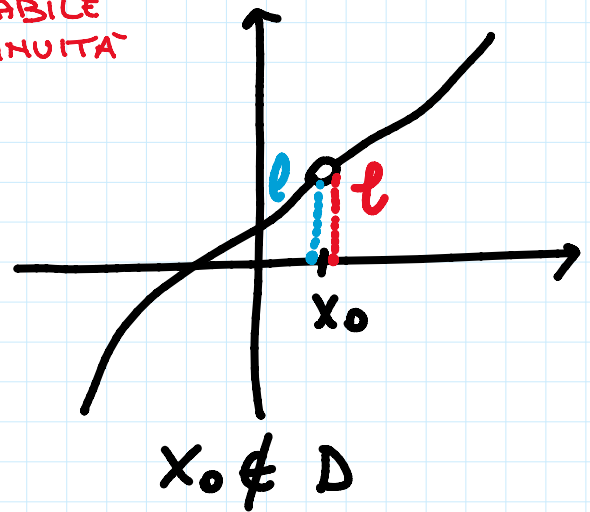
$X = 1$ AS. VERT. DX

$x = x_0$ DISCONTINUITÀ DI 2^a SPECIE SE
 ALMENO UNO DI $\lim_{x \rightarrow x_0^-} f(x)$ E
 $\lim_{x \rightarrow x_0^+} f(x)$ VALE ∞

DISCONTINUITÀ 3^a SPECIE: ELIMINABILE



OPPURE



$$\lim_{x \rightarrow x_0^-} f(x) = \lim_{x \rightarrow x_0^+} f(x)$$

SONO
 FINITI E
 UGUALI

MA

DIVERSI DA $f(x_0)$

1^o DISEGNO

O ADDIRITTURA

$x_0 \notin D$

2^o DISEGNO

IN QUESTI CASI

$f(x)$ PUO' ESSERE
 PROLUNGATA CON
 CONTINUITÀ E SI

COME $\overline{f(x)}$ SE $x \neq x_0$

SCRIVE $\bar{f}(x) = \begin{cases} f(x) & \text{se } x \neq x_0 \\ l & \text{se } x = x_0 \end{cases}$

ESAME

$$f(x) = \frac{\ln(1+2x^4)}{x^3(2^x-1)}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+2x^4)}{x^3(2^x-1)} =$$

$$\frac{\ln 1}{0} = \frac{0}{0} \text{ F.I.}$$

$$\Delta \begin{cases} A) x^3(2^x-1) \neq 0 \\ B) 1+2x^4 > 0 \end{cases}$$

$$A) x^3 \neq 0 \quad x \neq 0 \\ 2^x - 1 \neq 0 \quad x \neq 0$$

$$B) 1+2x^4 > 0 \quad \forall x$$

$$\boxed{\Delta \quad x \neq 0}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+f(x))}{f(x)} = 1$$

$$\boxed{f(x) \rightarrow 0}$$

$$\lim_{x \rightarrow 0} \frac{a^{f(x)} - 1}{f(x)} = \ln a$$

$$\boxed{f(x) \rightarrow 0}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+2x^4)}{x^3(2^x-1)}$$

$$= \lim_{x \rightarrow 0} \frac{\overset{1}{\underbrace{\frac{\ln(1+2x^4)}{2x^4}}_{\text{green}}} \cdot \overset{2x^4}{2x^4}}{x^3 \cdot \underbrace{\frac{(2^x-1)}{x}}_{\text{green}}} \cdot \overset{\ln 2}{\ln 2}}$$

$$\lim_{x \rightarrow 0} \frac{\cancel{2x^4}}{\cancel{x^4} \ln 2} = \frac{2}{\ln 2}$$

$$\bar{f}(x) = \begin{cases} f(x) & \text{se } x \neq 0 \\ l & \text{se } x = 0 \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{2} & \text{se } x = 0 \end{cases}$$

ESAME

$$f(x) = \frac{(1 - \cos 3x)^2}{x^2(1 - e^{9x^2})} \quad \mathbb{R} \setminus \{0\}$$

?

0

$$\lim_{x \rightarrow 0} \frac{1 - \cos f(x)}{(f(x))^2} = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{e^{f(x)} - 1}{f(x)} = 1$$

$$\lim_{x \rightarrow 0} \frac{(1 - \cos 3x)^2}{x^2(1 - e^{9x^2})} = \lim_{x \rightarrow 0} \frac{\left(\frac{1 - \cos 3x}{(3x)^2} \cdot (3x)^2 \right)^2}{x^2 \left(\frac{1 - e^{9x^2}}{9x^2} \cdot 9x^2 \right)}$$

$$\lim_{x \rightarrow 0} \frac{\left(\frac{1}{2} \cdot 9x^2 \right)^2}{x^2(-9x^2)} = \lim_{x \rightarrow 0} \frac{\frac{81}{4} x^4}{-9x^4} = \frac{81}{4} \cdot \left(-\frac{1}{9} \right) = -\frac{9}{4}$$

$$\bar{f}(x) = \begin{cases} f(x) & \text{se } x \neq 0 \\ -\frac{9}{4} & \text{se } x = 0 \end{cases}$$

PER VOI 13.06.2024

DETERMINARE $\bar{f}$ PROLUNGAMENTO
CONTINUO SU $\mathbb{R}$ DI

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$$f(x) = \frac{x \cdot \ln[1 + 16(x+1)^2]}{1 - e^{8(x+1)^2}}$$

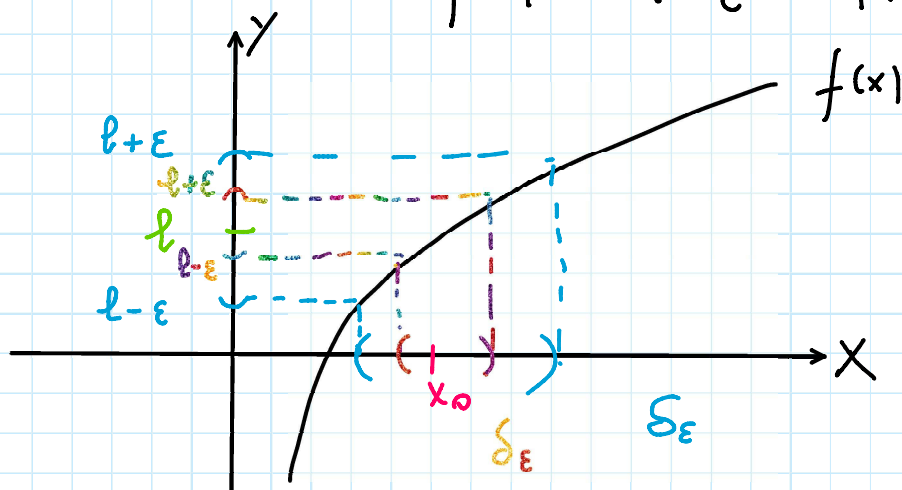
SUGGERIMENTO:

1) SICCOME NON HO x_0 DEVO FARE $\mathbb{D}$
E VERRA' $x \neq -1$

2) $\lim_{x \rightarrow -1} f(x)$ SICCOME $x \rightarrow -1$ CAMBIO VARIABILE
 $x \rightarrow -1$ ALLORA $x+1 \rightarrow 0$ PONGO
 $\boxed{x+1=t}$...

$$\lim_{x \rightarrow 1} \frac{x+1}{x+2} = \frac{2}{3} \text{ LO VOGLIO DIMOSTRARE}$$

DEF $\forall \varepsilon > 0 \exists \delta_\varepsilon \mid |x - x_0| < \delta_\varepsilon \Rightarrow |f(x) - l| < \varepsilon$



$$\lim_{x \rightarrow 1} \frac{x+1}{x+2} = \frac{2}{3}$$

IL LIMITE È DIMOSTRATO SE
"CON OGNI MEZZO A MIA DISPOSIZIONE"
L'ECITO" TROVO UN INTORNO
DI 1 PERCHÉ $x \rightarrow 1$

PARTO DA $\left| \frac{x+1}{x+2} - \frac{2}{3} \right| < \varepsilon \quad |f(x) - l| < \varepsilon$

$$\left| \frac{3x+3-2x-4}{3(x+2)} \right| < \varepsilon$$

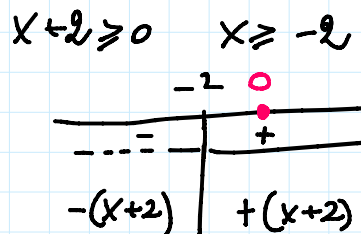
$$\left| \frac{x-1}{3(x+2)} \right| < \varepsilon$$

PROPRIETÀ $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$

$$\frac{|x-1|}{|3(x+2)|} < \varepsilon$$

LA VOGLIO TOGLIERE

$3(x+2)$ È POSITIVO
SE $x \geq -2$



$$\frac{|x-1|}{3(x+2)} < \varepsilon$$

$$\frac{|x-1|}{3(x+2)} < \varepsilon \quad \text{SE } x \geq -2$$

VOGLIO FARE UNA MAGGIORAZIONE.

PER $x \geq 0$ LA QUANTITÀ $3(x+2) > 3(0+2) = 6$

ALLORA

$$\frac{|x-1|}{3(x+2)} < \frac{|x-1|}{6}$$

L'IDEA È QUELLA DI
METTERE UN DENOMINATORE

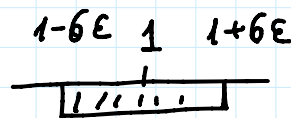
LAVORO SU

$$\frac{|x-1|}{6} < \varepsilon \quad |x-1| < 6\varepsilon$$

PIÙ PICCOLO
TOGLIENDO
X

$$-6\varepsilon < x-1 < 6\varepsilon$$

$$1-6\varepsilon < x < 1+6\varepsilon$$



$$\text{se } 1-6\varepsilon < x < 1+6\varepsilon \rightarrow \frac{|x-1|}{6} < \varepsilon$$

$$\rightarrow |x-1|$$

$$\rightarrow \frac{|x-1|}{3(x+2)} < \varepsilon \rightarrow \frac{1}{6} \varepsilon < \varepsilon$$

o

$$\lim_{x \rightarrow 1} \frac{3x+1}{x+3} = 1$$

$$\forall \varepsilon > 0 \exists \delta_\varepsilon \left| \text{se } |x-x_0| < \delta_\varepsilon \rightarrow |f(x)-l| < \varepsilon \right.$$

$$\left| \frac{3x+1}{x+3} - 1 \right| < \varepsilon$$

$$\left| \frac{3x+1-x-3}{x+3} \right| < \varepsilon \quad \left| \frac{2x-2}{x+3} \right| < \varepsilon$$

$$\text{se } x \geq 0 \quad x+3 \text{ È POSITIVA} \quad |x+3| = x+3$$

$$\frac{|2x-2|}{x+3} < \varepsilon \quad \text{PER } x \geq 0 \quad x+3 > 3$$

↑ PIÙ O AD X

$$\text{Allora} \quad \frac{|2x-2|}{x+3} < \frac{|2x-2|}{3} < \varepsilon \quad \text{m.c.m.}$$

$$|2x-2| < 3\varepsilon$$

$$-3\varepsilon < 2x-2 < 3\varepsilon \quad \text{SPOSTO IL -2}$$

$$2-3\varepsilon < 2x < 2+3\varepsilon \quad \text{DIVIDO PER 2}$$

OGNI TERMINE

$$1-\frac{3\varepsilon}{2} < x < 1+\frac{3\varepsilon}{2}$$

$$\delta_\varepsilon = \frac{3\varepsilon}{2}$$

PER VOI

$$\lim_{x \rightarrow 2} \frac{x+3}{x-4} = -\frac{5}{2}$$

"USATE $x \geq 0$ "

$$\lim_{x \rightarrow 1} \frac{x+1}{x+6} = \frac{2}{7}$$