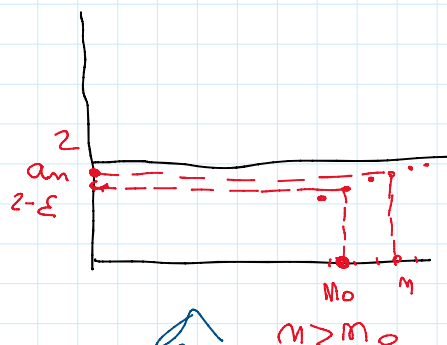
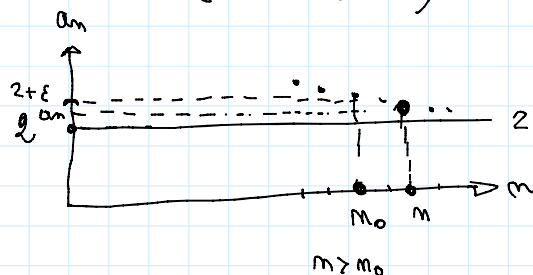


07. 01. 2020

DIMOSTRARE, USANDO LA DEFINIZIONE, CHE

$$\lim_{n \rightarrow \infty} \left(2 + \frac{5}{3^n} \right) = 2$$



UNICA DEFINIZIONE:

$$\forall \varepsilon > 0, \exists n_0 \mid |a_n - l| < \varepsilon \text{ se } n > n_0$$

1) VERIFICA PARTENDO DA $|a_n - 2| < \varepsilon$

$$a_n = 2 + \frac{5}{3^n} \quad l = 2$$

$$\left| 2 + \frac{5}{3^n} - 2 \right| < \varepsilon$$

$$\left| \frac{5}{3^n} \right| < \varepsilon$$

$$-\varepsilon < \frac{5}{3^n} < \varepsilon$$

$$\begin{cases} \frac{5}{3^n} < \varepsilon \\ \frac{5}{3^n} > -\varepsilon \end{cases} \quad \forall n \quad \frac{5}{3^n} > -\varepsilon$$

$$|f(x)| < k \quad (k \text{ POSITIVO})$$

$$\frac{-k}{k}$$

$$-k < f(x) < k$$

$$\Downarrow$$

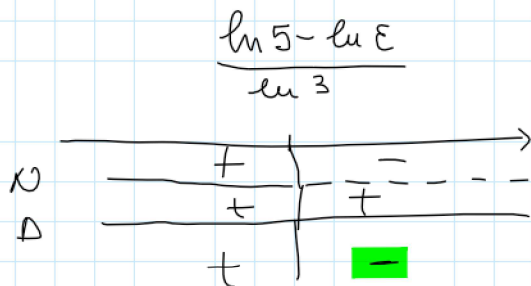
$$\text{SISTEMA} \begin{cases} f(x) < k \\ f(x) > -k \end{cases}$$

$$\frac{5}{3^m} < \varepsilon \quad \frac{5}{3^m} - \varepsilon < 0 \quad \frac{5 - \varepsilon \cdot 3^m}{3^m} < 0$$

TRATTO DISEQUAZIONE FRATTA. STUDIO SEGNO

$$+ \quad N > 0 \quad 5 - \varepsilon \cdot 3^m > 0 \quad -\varepsilon \cdot 3^m > -5 \quad \varepsilon \cdot 3^m < 5$$

$$D > 0 \quad 3^m > 0 \quad \forall m \quad 3^m < \frac{5}{\varepsilon}$$



USO ℓ_m

$$\sqrt[m]{\ln 3^m} < \ell_m\left(\frac{5}{\varepsilon}\right)$$

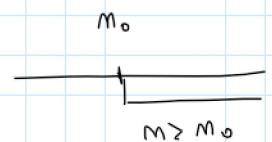
$$m \cdot \ln 3 < \ln 5 - \ln \varepsilon$$

$$m < \frac{\ln 5 - \ln \varepsilon}{\ln 3}$$

$N \cdot \varepsilon + m < \frac{\ln 5 - \ln \varepsilon}{\ln 3}$

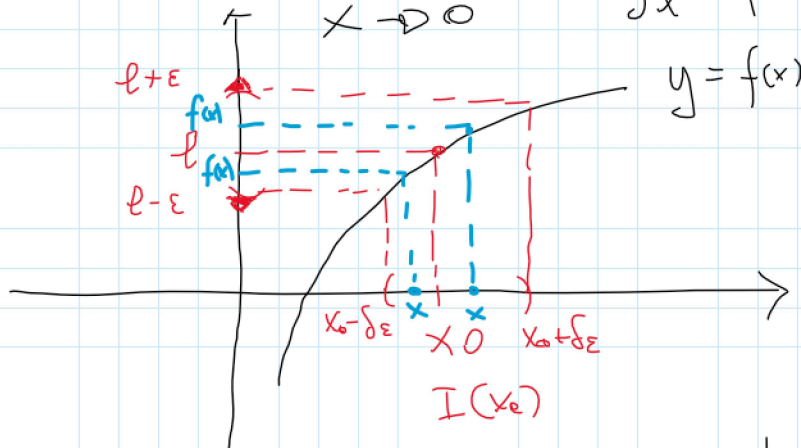
SOL $m > \frac{\ln 5 - \ln \varepsilon}{\ln 3}$

SE SCELGO $m_0 = \frac{\ln 5 - \ln \varepsilon}{\ln 3}$



07.01.2020

VERIFICARE $\lim_{x \rightarrow 0} \frac{1+2x}{3x-1} = -1$



$$\forall \varepsilon > 0, \exists \delta_\varepsilon > 0 \mid |f(x) - l| < \varepsilon \Leftrightarrow |x - x_0| < \delta_\varepsilon$$

TALE CHE

VERIFICA 1) PARTO DA $|f(x) - l| < \varepsilon$

2) TROVO UN INTERNO A x_0

$$\left| \overbrace{\frac{1+2x}{3x-1}}^{f(x)} - \overbrace{(-1)}^e \right| < \varepsilon$$

$$\left| \underbrace{\frac{1+2x}{3x-1} + 1}_{m.c.m.} \right| < \varepsilon$$

$$\left| \frac{\cancel{1}+2x+\cancel{3x}-\cancel{1}}{3x-1} \right| < \varepsilon$$

$$\left| \frac{5x}{3x-1} \right| < \varepsilon \Rightarrow -\varepsilon < \frac{5x}{3x-1} < \varepsilon$$

$$\begin{cases} \frac{5x}{3x-1} < \varepsilon & \text{I} \\ \frac{5x}{3x-1} > -\varepsilon & \text{II} \end{cases}$$

RISOLVO SINGOLARMENTE I e II e poi FACILIO L'INTERSEZIONE

I FRATTA $\frac{5x}{3x-1} - \varepsilon < 0$ m.c.m.

$$\frac{5x - 3\varepsilon x + \varepsilon}{3x-1} < 0 \quad \frac{(5-3\varepsilon)x + \varepsilon}{3x-1} < 0$$

• $N > 0$ $(5-3\varepsilon)x + \varepsilon > 0$ $(5-3\varepsilon)x > -\varepsilon$

mi devo
sincerare che questa
quantità sia POSITIVA
ALTRIMENTI
devo CAMBIARE
SEGNO E VERSO

$\varepsilon \approx 0,1$
 $5-3\varepsilon$
POSITIVA

$$x > \frac{-\varepsilon}{5-3\varepsilon}$$

$$\frac{-\varepsilon}{5-3\varepsilon} \quad \frac{1}{3}$$

N	-	-	+	+
D	-	-	-	+
	+	+	-	+

• $D > 0$ $3x-1 > 0$ $x > \frac{1}{3}$

$$\text{I} \quad \left| \frac{-\varepsilon}{5-3\varepsilon} < x < \frac{1}{3} \right|$$

II FRATTA

$$\frac{5x}{3x-1} > -\varepsilon \quad \frac{5x}{3x-1} + \varepsilon > 0$$

$$\frac{5x + 3\varepsilon x - \varepsilon}{3x-1} > 0$$

$$\frac{(5+3\varepsilon)x - \varepsilon}{3x-1} > 0$$

$$\bullet N > 0 \quad (5+3\varepsilon)x - \varepsilon > 0 \quad (5+3\varepsilon)x > \varepsilon \quad x > \frac{\varepsilon}{5+3\varepsilon}$$

$$\bullet D > 0 \quad 3x-1 > 0 \quad x > \frac{1}{3}$$

$$\frac{\varepsilon}{5+3\varepsilon}$$

$$\frac{1}{3}$$

N
D
N/D

$$\text{I} \quad \left| x < \frac{\varepsilon}{5+3\varepsilon} \vee x > \frac{1}{3} \right|$$

$$-\frac{\varepsilon}{5-3\varepsilon} \quad \frac{\varepsilon}{5+3\varepsilon}$$



$$-\frac{\varepsilon}{5-3\varepsilon} < x < \frac{5}{5+3\varepsilon}$$

I(0) $x \rightarrow 0$
LIMITE VERIFICATO

TAYLOR

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^6}{6!} + \dots$$

CALCOLARE IL LIMITE SEGUENTE (DE L'HOPITAL)

$$\lim_{x \rightarrow 0} \frac{-2x - 3 + 3(1+x)^{\frac{2}{3}} \cdot 1}{x^2 \cdot (1+x)^{\frac{1}{3}}} = \frac{0}{0} \quad H$$

$$\lim_{x \rightarrow 0} \frac{-2 + 3 \cdot \frac{2}{3} (1+x)^{\frac{2}{3}-1} \cdot 1}{2x \cdot (1+x)^{\frac{1}{3}} + x^2 \cdot \frac{1}{3} (1+x)^{\frac{1}{3}-1} \cdot 1}$$

$$\lim_{x \rightarrow 0} \frac{(-2) + 2(1+x)^{-\frac{1}{3}}}{2x \cdot (1+x)^{\frac{1}{3}} + \frac{1}{3} x^2 (1+x)^{-\frac{2}{3}}} = \frac{0}{0} \quad (H)$$

$$\lim_{x \rightarrow 0} \frac{2 \cdot (-\frac{1}{3})(1+x)^{-\frac{1}{3}-1}}{2(1+x)^{\frac{1}{3}} + 2x \cdot \frac{1}{3} (1+x)^{\frac{1}{3}-1} + \frac{2x}{3} (1+x)^{-\frac{2}{3}} + \frac{1}{3} x^2 (-\frac{2}{3})(1+x)^{-\frac{2}{3}-1}} = \frac{-\frac{2}{3}}{2} = -\frac{2}{3} \cdot \frac{1}{2} = -\frac{1}{3}$$

USO DI TAYLOR

- 1) INIZIO A SVILUPPARE I DENOMINATORI E MI FERMO IL PRIMA POSSIBILE A PATTO CHE QUANDO CAMBIO IL DENOMINATORE ESSO NON VENGA IDENTIFICATO ZERO IN QUESTO MODO OTTENGIO L'ORDINE DEL DENOMIN.
- 2) SVILUPPO IL NUMERATORE CHE COMPLESSIVAMENTE DOVREBBE ARRIVARE ALLO STESSO ORDINE DEL

DENOMINATORE. MAI ALL'ORDINE INFERIORE

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^2 \cdot \sin x}$$

mi fermo ad x $\mathcal{D}: x^2 \cdot x = x^3$

1) DENOMINATORE $\sin x = x - \frac{x^3}{3!} \text{ ecc} \dots$ POSSO FERMARMI SUBITO AD x

PERCHÉ $\mathcal{D}: x^2 \cdot x = x^3 + o(x^3)$

L'ORDINE DEL DENOMINATORE È 3

SONO GLI ALTRI TERMINI DELLO SVILUPPO CHE LO OMETTO E VALGONO A ZERO PIÙ VELOCEMENTE DI x^3
 $o(x^3) = x^4, x^5, x^6, \dots$

2) SVILUPPO N CHE COMPLESSIVAMENTE DEVE ARRIVARE A ORDINE 3

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots$$

$$H = \sin x - x \rightarrow \left[x - \frac{x^3}{3!} \right] - x = -\frac{x^3}{6} \quad \text{NUTTI ORDINE 3}$$

$$\lim_{x \rightarrow 0} \frac{-\frac{x^3}{6}}{x^3} = -\frac{1}{6}$$

$$\lim_{x \rightarrow 0} \frac{\sin(2x) - \ln(1+x)}{\cos\left(\frac{x}{2}\right) - 1} \stackrel{\textcircled{2} \text{ VA DAVANTI}}{=} \lim_{x \rightarrow 0} \frac{\sin(2x) - 2\ln(1+x)}{\cos\left(\frac{x}{2}\right) - 1}$$

DENOMINATORE PER LO SVILUPPO USO "t"

$$\cos t = 1 - \frac{t^2}{2!} + \frac{t^4}{4!} \dots$$

$$\cos\left(\frac{x}{2}\right) = 1 - \frac{\left(\frac{x}{2}\right)^2}{2!} + \frac{\left(\frac{x}{2}\right)^4}{4!} + \dots$$

NUM : $\cos\left(\frac{x}{2}\right) - 1$ NON POSSO, AL $\cos\left(\frac{x}{2}\right)$ FERMAMI AD 1,
ALTRIMENTI HO $1 - 1 = 0$

NUM $1 - \frac{x^2}{4} \cdot \frac{1}{2} - 1$ MI FERMO A $1 - \frac{x^2}{2}$

NUM = $-\frac{x^2}{8} + o(x^2)$ ORDINE È 2

NUMERATORE: COMPLESSIVAMENTE CERCO ORDINE 2

$\sin 2x - 2 \ln(1+x)$

$\sin t = t - \frac{t^3}{3!} + \dots$ $\ln(1+x) = x - \frac{x^2}{2} + \dots$

$\sin(2x) = 2x - \frac{(2x)^3}{3!} + \dots$ RICORDA $2(\ln(1+x)) = 2\left(x - \frac{x^2}{2} + \dots\right)$
 $= 2x - x^2 + \dots$

$\sin 2x - 2 \ln(1+x) =$
 $2x - (2x - x^2) = \cancel{2x} - \cancel{2x} + x^2 + o(x^2)$

$\lim_{x \rightarrow 0} \frac{x^2}{-\frac{x^2}{8}} = -8$

$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x^2 \cdot \sin \frac{1}{x}}$

NO TAYLOR $\sin\left(\frac{1}{x}\right)$ per $x \rightarrow 0$ $\frac{1}{x} \rightarrow \infty$ $\sin \infty$

NO ASINTOTICITA' PER IL SENO

$\sin t \sim t$ $t \rightarrow 0$

NO DE L'HOPITAL: SE DERIVO $\frac{1}{x}$ OTTENG $-\frac{1}{x^2}$ AUMENTO IL GRADO

$$\lim_{x \rightarrow \infty} \frac{\ln(1+x)}{x^2 \cdot \sin \frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{x}{x^2 \cdot \sin \frac{1}{x}}$$

$$\ln(1+x) \sim x$$

$$\lim_{x \rightarrow \infty} \frac{1}{x \sin \frac{1}{x}}$$

PROBLEMA $\lim_{x \rightarrow \infty} x \cdot \sin \frac{1}{x}$

TEO CONTRASTO

$$0 \leq \left| \underbrace{x}_{\leq 1} \cdot \underbrace{\sin \frac{1}{x}}_{\leq 1} \right| \leq |x| \cdot \left| \sin \frac{1}{x} \right| \leq |x| \cdot 1 = |x|$$

$$0 \leq \left| x \sin \frac{1}{x} \right| \leq |x| \quad \text{per } x \rightarrow 0$$

$$0 \leq \left| x \sin \frac{1}{x} \right| \leq 0 \Rightarrow \text{per } x \rightarrow 0 \quad x \cdot \sin \frac{1}{x} \rightarrow 0$$

$$\lim_{x \rightarrow 0} \frac{1}{x \sin \frac{1}{x}} = \frac{1}{0} = \infty$$

f(x) LIMITATA $|f(x)| < M$
 $g(x) \rightarrow 0$

$$0 \leq |f(x) \cdot g(x)| \leq |g(x)| \cdot M$$

$$0 \leq |f(x) \cdot g(x)| \leq 0 \quad g(x) \rightarrow 0$$